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A Probabilistic Theory of Supply and Demand

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Abstract
We propose a model of supply and demand that replaces the conventional deterministic model with one based on a normal probability distribution. This suggests that prices and quantity realizations fluctuate without any fundamental changes in the underlying variables. Thus, variability becomes an inherent feature of the system in place of static equilibrium.

Keywords: Law of supply and demand, probabilistic demand, probabilistic supply, endogenous fluctuations of prices and quantities, fuzzy logic, normal distribution

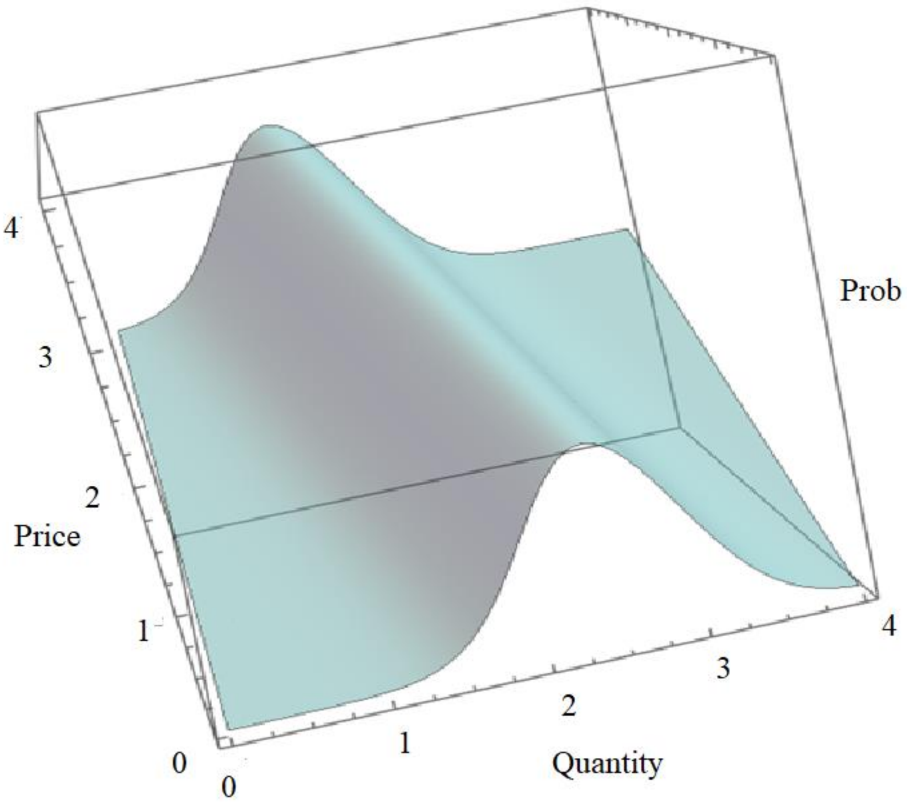
JEL: A10; A20; B41; B50; D01

The concepts of supply and demand are one of the fundamental building blocks of economics. They describe the interaction of a downward sloping demand function and an upward sloping supply function. The assumption underlying their interaction is deterministic: at every price there is a unique quantity of a good demanded [Q_d=f(p)] by an individual consumer and supplied [Q_s=f(p)] by a firm, ceteris paribus. When these values are aggregated the deterministic market equivalent is obtained.

We now relax this deterministic view and frame the supply and demand problem in terms of the kind of radical uncertainty reflected in the quantum-theoretical view of reality (Farjoun and Machover 1983). In contrast to the classical conception of the world, physicists realized at the turn of the twentieth century that elementary particles do not have the same physical properties as objects on a human scale. Based on empirical evidence, giants like Albert Einstein, Max Planck, Niels Bohr, and many others discovered that physical laws that describe macroscopic objects cease to be valid at the subatomic level where particles do not have deterministic attributes. Rather, they have ambiguous characteristics, are probabilistic in nature, and there are real limits to how accurately their properties can be predicted prior to measurement. In general, one cannot predict with certainty what will happen at the subatomic level, but only provide probabilities.

Regrettably, this revolution in epistemology still eludes mainstream economics which continues to conceive of the behavior of its fundamental units, human economic agents, as being deterministic. In the current context, the rejection of the deterministic view of human behavior implies that one cannot know with certainty the quantity that will be purchased of a good by an individual at the current price (per unit of time) let alone at any future price before its actual realization (Mantegna and Stanley, 2000). This indeterministic view of demand is not a question of measurement error; rather, like Heisenberg's uncertainty principle, it is rooted in the intrinsic limitations of human knowledge, a basic feature of which is the uncertainty associated with human action: the quantity that will be demanded at any price is vague, uncertain, and depends on a probability distribution (Kahneman, Sibony, and Sunstein 2021).[1] The justification is that consumers do not know themselves with any degree of certitude how many apples or oranges, or any other product for that matter, they will consume at its current price (per unit time) until the moment the purchase takes place, let alone in the future if that price (or other prices) were to change. Wants are “fuzzy” since consumers are unsure, hesitate, are subject to imperfect information, must decide within a time constraint using judgement, or err (Zadeh, 1975; Faber, Manstetten, and Proops, 1992). Similarly to quantum mechanics, the price-quantity pair is not knowable ex-ante, in principle, until observed.

Figure 1: Probabilistic Demand Function Given by a Normal Distribution.



For the sake of simplicity, we assume that at every price the consumer’s cumulative probability of choosing a product can be described by a normal distribution with a standard deviation σ and mean μ:

$$Q_d = N(\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}[\frac{q-\mu}{\sigma}]^2} \qquad \text{(Figure 1).}$$

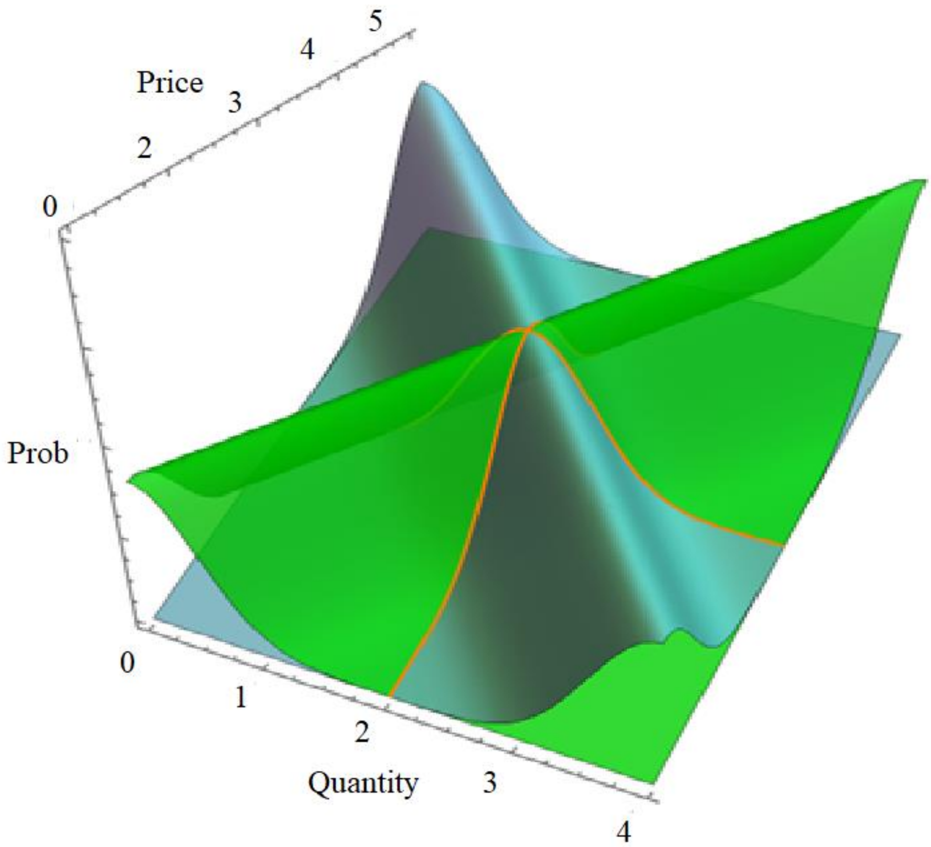
This bears some similarity to the probabilistic framework proposed by McFadden in the case of choice between two products.[2] He assumed that “utility is a random function,” (1980, p. S14). The sources of

[1] This is similar to econophysics whose “approaches to traditional economic problems are essentially probabilistic (Cockshott et al., 2009). For another stochastic approach that does not impose the normal distribution see Cerreia-Vioglio et al., (2022).

[2] For a probabilistic framework modeling bid and ask stock market prices see (Biais, Hillion, Spatt. 1995).

randomness in his view "are unobserved variations in tastes and in the attributes of alternatives, and errors of perception and optimization by the consumer" (1980, p. S15). Deaton suggests similarly that "the absence of complete price information causes mistakes to be made" by consumers (Deaton 1977, p. 900). This can also be thought of as "semi-rational" behavior in a random utility model with preferences subject to unknowable "probability mechanisms" which might be considered stochastic (Luce and Suppes, 1965, p. 256). For the sake of simplicity, we impose the normal distribution on this randomness, and further assume that σ does not vary with price D(p)=N(μ,σ). The envelope of μ traces the conventional demand function (for instance: μ=a-bp) (Figure 1). Clearly, these simplifying assumptions can all be relaxed in subsequent analysis.

Figure 2: Probabilistic Demand and Supply Functions, Both Given by a Normal Distribution.



The supply is specified similarly since producers also face multidimensional uncertainty regarding how much to produce at various prices. This is not conceptualized as the conventional shocks in macroeconomics (Blanchard, 1989). Instead, the fluctuations are inherent in the Knightian uncertainty characteristics of production (Knight, 1921). Much depends on the evolution of inventory levels over time, how long production takes to replenish inventories, the extent to which the product is perishable or can go out of fashion, actions of competitors, expectations regarding the macroeconomic environment, demand for the product the last period, and a number of unknowable factors such as supply chain interruptions.

The intersection of the stochastic supply and demand functions yields the set of price-quantity realization pairs (Figure 2). The realizations in this model become the "intersection" of the product of two normal bivariate distributions (expressed as a product):

$$z = -(\exp(-(x-y)^{2} * \exp(-(x-y)^{2}))).$$

Figure 3 displays this set or realizations with 1,000 random price-quantity pairs drawn from the intersection of two normal probability distributions each with mean (μ) = 2 and standard deviation (σ) = 0.5.[3] The height of the cone (on the z-axis) represents the probability of obtaining a particular pair.

Figure 3: The Probability of a Price-Quantity Pair Is Given by the Intersection of Supply and Demand with μ = 2 and σ = 0.5.

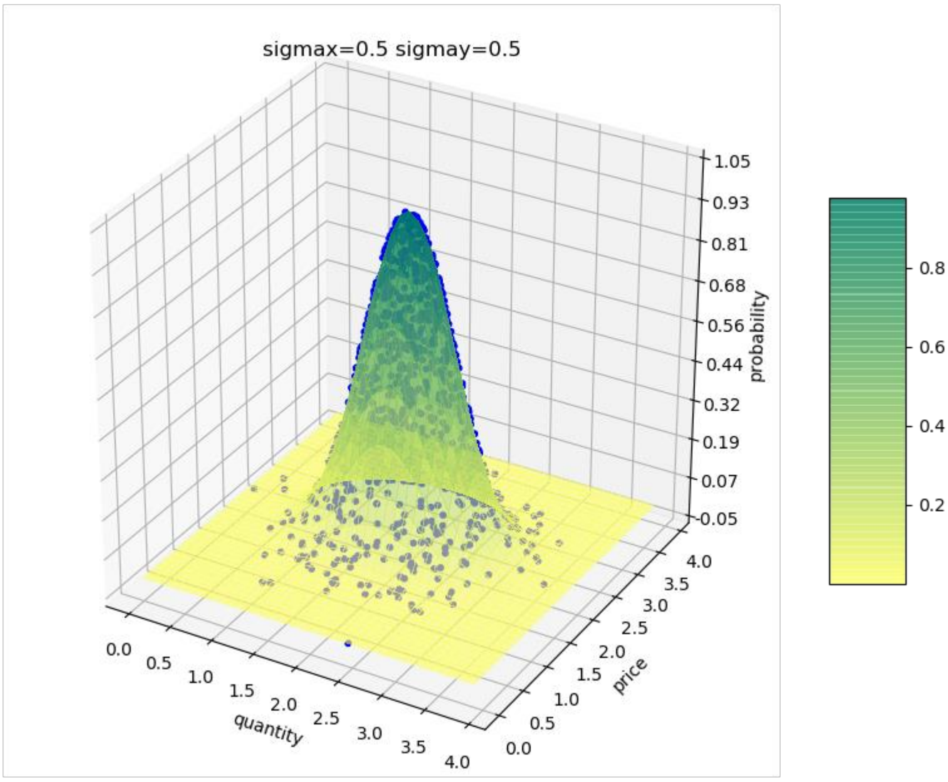
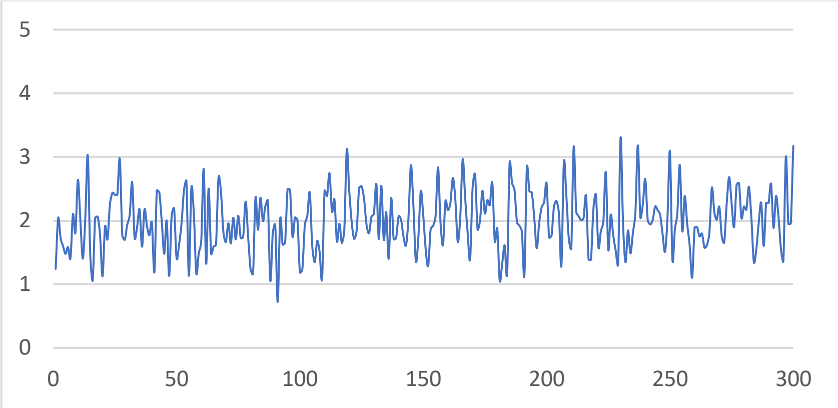


Figure 4 depicts the view of this set seen from the top down, i.e., projected on the x,y plane, showing that the values are bounded within a fuzzy circle that concentrate around the mean values.

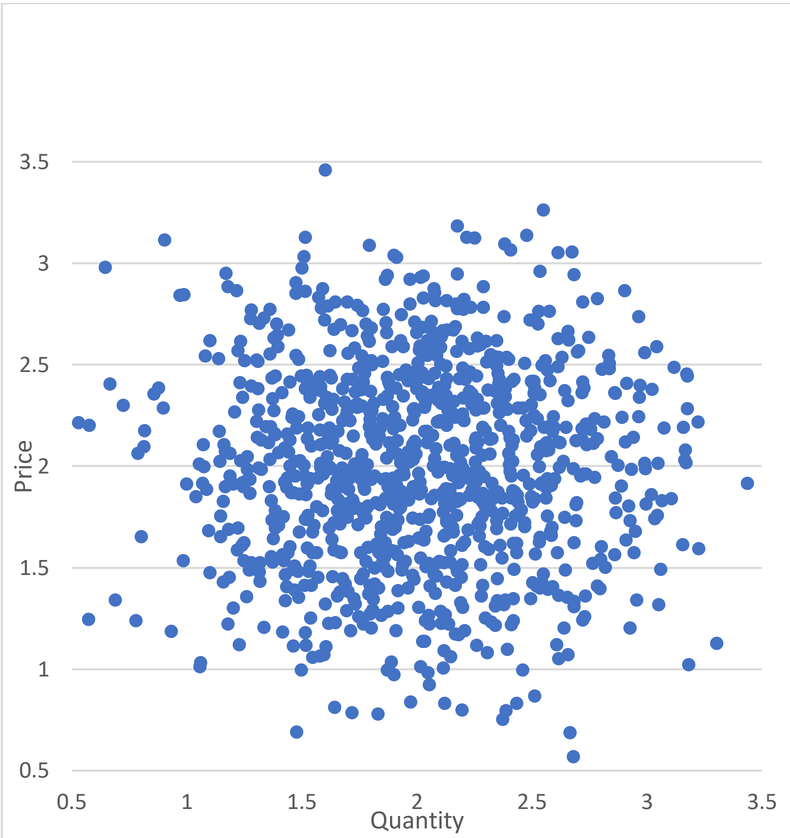
Figure 4: Random Price Fluctuations μ = 2, σ = 0.5

[3] Since the sum of N independent normal distributions has a mean of $N\mu_i$ and a variance of $N\sigma_i^2$ for 1000 consumers the market demand would have a mean of 2000 and its σ would become 15.8 and the coefficient of variation would decline from 0.25 to 0.0079.



Price fluctuates randomly, although the fundamentals of demand and supply remain unchanged, i.e., neither income, tastes, production costs, or anything else change (Figure 5).

Figure 5: View of the Probabilities from Top Down, with μ = 2, σ = 0.5 for both functions.



In Figure 6 the supply curve's σ is decreased to 0.25, indicating how the bound of a set of probable equilibrium values becomes an elliptic cone and its projection on the x,y plane becomes an ellipse (Figure 7).

Figure 6: The Probability of a Price-Quantity Pair Is Given by the Intersection of Supply and Demand with μ =2; σ(price)=0.5 and σ(quantity)=0.2.5.

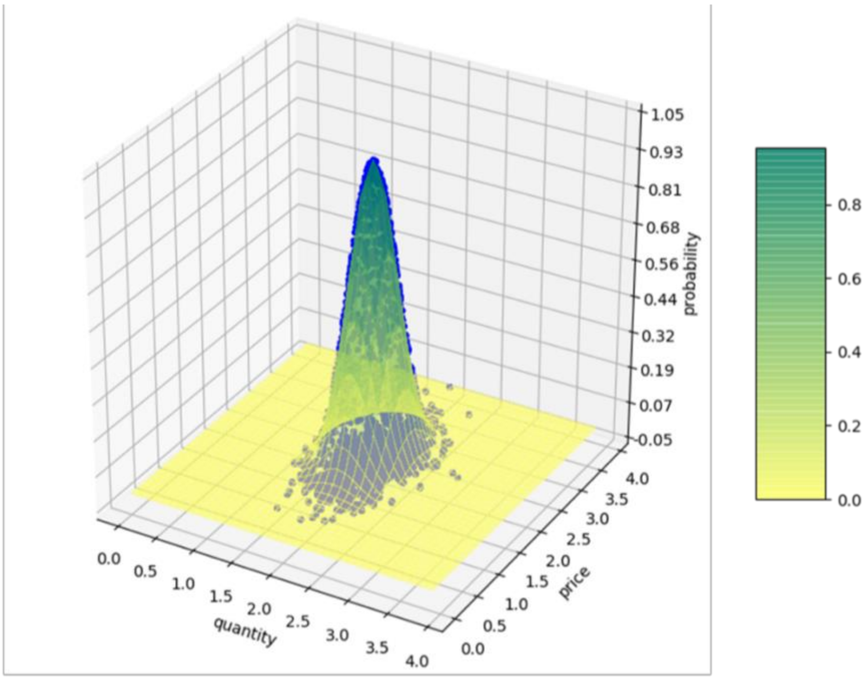
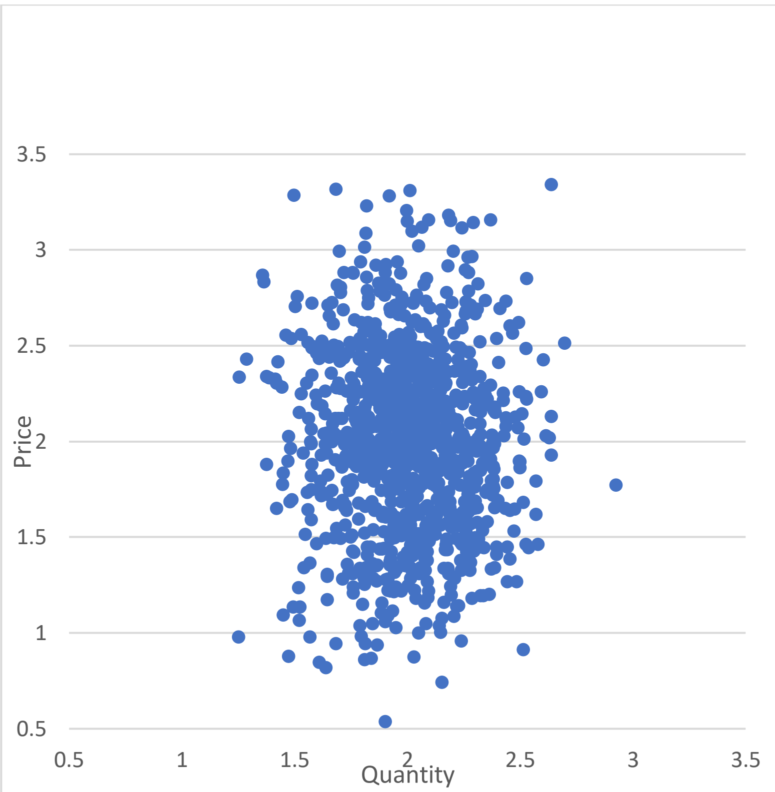


Figure 7: Realizations in Two Dimensions, with μ =2; σ(price)=0.5 and σ(quantity)=0.2.5.



In the conventional deterministic framework, the supply and demand curves intersect in a point and produce an equilibrium value. There is no randomness in that model. The consumers know exactly how much they demand, and the market demand is the just the sum of individual demands, so it is also a point. Randomness might emerge in measurement, but the theoretical model is itself deterministic. In contrast, in the present probabilistic framework there is no such equilibrium at the individual level and market outcomes are also probabilistic, i.e. depend on a probability distributioin. Instead, there is a fuzzy set of realized price-quantity pairs. Hence, price-quantity pairs fluctuate within a range, determined by the standard deviations of the respective functions' probability distribution, but this randomness emerges without any external changes in the supply and demand functions, i.e., without changes in costs, or incomes, or utility functions. Thus, in this model price variations are endogenous to the system.

This model is a falsifiable hypothesis requiring further research. Obviously, individuals could be surveyed to see how much of some items they will demand under certain conditions and then see how accurate the realizations were compared to their expectations. Moreover, the above model points to a general way of thinking about other economic phenomena in order to overcome the tendency to conceptualize the economy using deterministic theories. An extreme example of the misplaced use of deterministic models is the compulsive insistence on relying on microfoundations in order to understand aggregate macroeconomic phenomena (Stiglitz, 2018). This is precisely the kind of fundamental error that quantum physicists began to remedy well over a century ago. Such a paradigm shift would be advantageous in economics as well.

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