An Intersubjective Theory of Value

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Abstract

Every quantitative order has a structure. These structures vary greatly between different orders, for example, those of mass, temperature and probability. Economics has neglected to investigate the structure of exchange-value, and this paper seeks to identify it. By considering the structural implications of several market phenomena, including negatively sloped demand curves, inelastic demand, monetary inflation and the commensurability problem with units of account, this paper demonstrates that exchange-value is a boolean algebra. This result is considered in the light of the "homogeneity problem" in GET and of Becker's "scarcity principle".

1. Introduction

The subjective theory of value holds that the *quantitative order* variously called "exchange-value", "market-value" and "money-value" is analyzable into *intra*subjective preferences and ratios of exchange between pairs of commodities. This paper will demonstrate:

1. that this presumed reducibility is logically impredicative;
2. that exchange-value is conditional upon and describes a broad *inter*subjective space;
3. that the structure of exchange-value is boolean rather than euclidean;
4. that this boolean structure, rather than the maximizing behavior of individual agents, accounts in the main for downward slopping demand curves; and
5. that some well-known paradoxes that appeared in twentieth-century economic theory were due to applying euclidean and intrasubjective assumptions to intersubjective boolean reality.

2. The Nature of the Beast

Length, time, mass, angular distance, probability, temperature and other measurement orders are characterized by various properties which together constitute a *structure*. These properties are open to description by elementary abstract algebra. That the metrical properties of these measurement orders -- for example, mass, temperature and probability -- diverge widely stands as common knowledge. So too does the fact that the Einsteinian revolution centered upon a reassessment of the metrics of fundamental orders of physical measurement. Previously, all physicists had assumed that lengths and masses could, in principle, be combined indefinitely in a manner analogous to arithmetical operations on the set of real numbers. In economics a similar euclidean assumption structures the conception of market or exchange value. As with one's everyday perceptions of the physical world, economists take this euclidean assumption so much for granted that usually it escapes mention. In theoretically fundamental texts, however, one does find this, *the* most central structural hypothesis of economic theory, explicitly stated. Debreu, for example, launches his *Theory of Value* by noting that his analysis is "organized around the concept of a price system or, more generally, of a value function defined on the commodity space," and whose euclidean structures he makes explicit in the course of the work. [ix] [1] More profound, however, is what Debreu fails to do. His demonstration of a euclidean structure extends only to his *concept* of "exchange-value". Regarding exchange-value itself, that is, whether his concept correctly describes it, he offers no evidence.

Debreu's choice, and that of economics generally, to disregard the possibility that one's most basic presuppositions might not correspond to reality, is, as noted, part of a once venerable tradition. Its greatest champion was Kant. In *The Critique of Pure Reason* he argued that it is possible to know some of reality's basic structures *a priori*, and he used everyday euclidean notions of space and time as his central example. History, however, is cruel to *a priorism*, and never more so than with Kant's celebrated pronouncements on space and time. But, as evidenced by the acclaim attached to Debreu's and similar essays, economics has not yet escaped from subjective idealism as a way of thinking. Economists, with exceptions, do not permit empiricism to come between them and their basic concepts.

Yet that is what I propose to do with respect to "exchange-value". The parallel between the distinction that the following passage from Bertrand Russell describes and the one that this paper is going to pursue is fairly exact.

> 'Geometry', as we now know, is a name covering two different studies. On the one hand, there is pure geometry, which deduces consequences from axioms, without inquiring whether the axioms are 'true'; this contains nothing that does not follow from logic, and is not 'synthetic' . . . On the other hand, there is geometry as a branch of physics, as it appears, for example, in the general theory of relativity; this is an empirical science, in which the axioms are inferred from measurements, and are found to differ from Euclid's. [688]

Economists, like physicists of an earlier age, share the general human weakness of presuming that the world's deep structures resemble surface appearances. For economics this means believing that everyday, commonsense understanding suffices for comprehending exchange-value's structure. Leaving this most fundamental of questions in the realm of pre-science or commonsense also gives the presumed answer the epistemological certainty demanded by economics' theoretical heritage. Neoclassical value theory defines both general and partial equilibrium as a relation between market supply and demand functions, and it defines these functions as summations of individual supply and demand schedules. The postulation of these summations requires the assumption of *additivity*, the key feature of the euclidean structure. Additivity, however, "is only valid if the demand functions of the various individuals are independent of each other." [Morgenstern 1948, p.175] Independence means no *intersubjective effects*, that is, that each agent's demand, rather than being affected by the demands of other agents, is purely *intra*subjective. It is only in this special case that, by definition, the neoclassical theory of value pertains. This requirement of additivity, especially of demands, locks neoclassicalism and economics generally into presuming that exchange-value is euclidean. Recognition of the importance of intersubjective supply and demand phenomena in determining market outcomes frees economics from this traditional presumption, thereby making psychologically possible an empirical and critical inquiry into the structure of exchange-value. This paper seeks to initiate this project.

3. The Structure and Methodology of this Paper

The argument develops in three stages. The first entertains a few fundamental questions about the minimum requirements for the establishment of exchange-value as a quantitative order. Tradition says exchange-value is, like weight and probability, a relational property -- not one that holds for commodities individually as do mass and extension for physical objects. But what exactly are the terms of the exchange-value relation? And how many are required for its existence? The accepted view that exchange-value is reducible to a relation between market agents and two commodities will be tested for logical coherence. The second and main stage of the argument considers what various commonly observed market phenomena infer about the structure of exchange-value. Is this empirical knowledge consistent with the view that exchange-value is euclidean, and, if not, is it possible to identify its real structure? The third stage considers the broad implications of the results of stage one and two, and shows that several major theoretical anomalies that surfaced in the twentieth century result from economics' misunderstandings of the structure of exchange-value.

Four aspects of this paper's methodology need foregrounding. First, the analysis is purely *synchronic*. Time, including the hypothetical instantaneous time of equilibrium analysis, does not figure.

Second and more important, the method is *retroduction*. [Lawson, 1997] Economists accustomed only to working with the deductive method -- as described by Russell above -- will be predisposed to read this paper inside-out, making it appear that I have indulged in a bizarre selection of axioms to produce a whimsical Escher-like landscape. But such an error is easily avoided by grasping one crucial point. *This paper's logical inferences proceed from the level of observable phenomena back to the level of underlying structures or "axioms" in Russell's second sense of the word.* The goal is to discover, not the properties of an axiom system, but rather those of a particular real-life system of intersubjective relations, the market price system. In lieu of economics' traditional axioms, the argument begins on the basis of a presumption of possibility, namely, that, just as the objective character of lightning is not exhausted by its visual appearance, so it is unlikely that the structure of exchange-value reveals itself fully in the situation of everyday market exchange. Given the long hegemony of *a priorism* in this field, little, in fact, is known about the metrical structure of exchange-value. This paper will show that its structure is more highly defined than previously thought.

Third, *structure*'s explanatory significance must be clarified. As noted above, structure plays a foundational role in neoclassical explanation. But because neoclassicalism's euclidean assumption performs its function unnoticed, it has been possible to foster the myth that neoclassical conclusions have been reached solely on the basis of axiomatic generalizations about individual behavior. If inquiry reveals that exchange-value is not euclidean, then the theorizing of value must begin afresh. A logical starting place would be the possibility that some of the facts explained by neoclassical axiomatics, (i.e., by "a set of postulated behavioural regularities") are, in reality, determined *directly* by exchange-value's non-euclidean structure. [Lawson 1998, p. 102]

Fourth, as previously noted, the belief that exchange-value is euclidean is folkloric. This paper breaks with that tradition, taking the question of exchange-value's structure out of the realm of folklore and into the realm of science. This means looking for and at evidence as to exchange-value's structure. Doing so requires a bit of very elementary abstract algebra. Its use here, however, will be kept to a minimum and paralleled with verbal explanations. Only once, and then very briefly, will the mathematics lift slightly above this basic level. Here, as elsewhere, the abstract account will be matched by a verbal one.

4. Testing the Notion of Comparative Exchange-Value for Logical Coherence

Physics' classical concepts of length and mass emerge from comparative concepts, pairs of empirically defined relations, one equivalence, the other precedence, which have been shown to hold between pairs of objects.[2] Can exchange-value also be identified as originating with or shown to be reducible to a set of relations between a pair of objects, in other words, to a concept of comparative exchange-value? The following passage from Alfred Marshall identifies exchange-value as, like mass and length, based on a comparative concept, one holding between pairs of commodities.

> The value, that is the exchange value, of one thing in terms of another at any place and time, is the amount of that second thing which can be got there and then in exchange for the first. Thus the term value is relative, and *expresses the relation between two things* at a particular place and time. [emphasis added] [*Principles* 1920, p. 61]

To give it every possible chance, I want to formulate this notion of comparative exchange-value more precisely before seeing if it holds up to logical analysis. Thus:

> For pairs of commodities, there is the exchange-value of each commodity *relative to the other*, in the sense that quantities of the two commodities are said to be equal in value if they exchange for each other and to change in value if there is a change in the pair's market-clearing exchange ratio.

This reformulation, as well as Marshall's sentence, appears coherent. But the twentieth century discovered that the logical relations of statements are not always what they appear to be. So I am going to test the stated notion of comparative exchange-value against the general principle that, between any two magnitudes of the same empirical order, an equality relation either holds or does not. Consider two commodities X and Y, and whose units are x and y. Let a, b, and σ be rational positive numbers.

Assume that the initial market-clearing ratio of $ax:by$ changes to $ax:\sigma by$. Then, according to the concept of comparative exchange-value, the exchange-values of quantities of X relative to Y have changed. Any two quantities of the same order are either equal or not equal. Therefore,
the exchange-value of σby relative to X at the new exchange ratio is either equal or not equal to the exchange value of by at the old exchange ratio.

First assume that it is equal. Then, because at the old ratio the exchange-values of ax and by were equal and at the new ratio the exchange-values of ax and σby are equal, it follows that the exchange-value of ax is unchanged. This contradicts the assumption that the exchange-values of quantities of X relative to Y have changed, and so one must conclude that this case cannot obtain.

Assume the other possibility: the exchange-value of σby at the new exchange ratio is not equal to the exchange-value of by at the old exchange ratio. If, relative to X, by and σby are not equal in exchange-value, then by the concept of comparative exchange-value they do not exchange for the same number of units of X. However, by assumption they do exchange for the same number of units of X. Therefore, this case also cannot obtain. And this exhausts the logical possibilities.

The concept of comparative exchange-value generates paradoxes because it is circular. It defines a commodity's exchange-value in terms of the exchange-value of a second commodity whose exchange-value is defined in terms of the exchange-value of the first. In technical terms, this constitutes "vicious circularity" which renders the definition impredicative.

This simple but unexpected outcome of the test for logical coherence shows that, as a quantitative order, exchange-value has unexpected properties. Already the euclidean presumption appears farfetched. The initial result also bears directly on this collection's theme. Whereas the traditional but impredicative notion of exchange-value reduced to the field of two atomistic agents -- two independent subjectivities -- making an exchange, we now know that the minimal field for existence of exchange-value must be larger. It is premature to ponder what this might mean. Doing so now would risk perplexity. First we must pursue other possibilities for analysis and hope that with further results all will become clear.

5. False Resemblances

Confusions, like the one unearthed in the previous section, come easily when thinking about exchange-value because in two respects it bears a false similarity to familiar physical magnitudes. First, the notion of exchange-value as a relation between two commodities exhibits a superficial resemblance to comparative concepts of mass and length. These physical concepts, however, are not predicated as relations between individual masses and lengths. It is only their measurement numbers that are conceived in this way. Instead, Newtonian physics predicates mass and extension as properties possessed by bodies independently of their relations to other bodies. This independence saves concepts of comparative length and mass from impredicativeness. [Carnap,1966, pp. 51-61)

Second, and related to the first, although exchange-value numbers are expressed on a ratio scale like mass and length numbers, they are generated in a profoundly different manner. Physical measurement numbers refer to physical phenomena, called concrete quantities, which have been found to have a structure isomorphic to the system of units and numbers (abstract quantities) by which they are represented. A cardinal point is that these concrete physical quantities do not come into being as the result of humankind's invention of processes of numerically representing them. If a means of numerically representing the weight of your body had never been invented, you would experience its weight all the same. The existence of the properties of extension and mass are independent of the processes by which they are measured or compared. In contrast, the quantitative order of exchange-value does not exist independently of the process which assigns exchange-value numbers. Without market exchange there is no exchange or market value. Market exchange, in other words, is the process by which the exchange-value order, not just the numbers which describe it, comes into being.

The fact that the process that determines concrete exchange-values also assigns numbers to represent them invites conflation of concrete exchange-values and exchange-value numbers. The latter, stripped of their units, belong to *R*, the set of positive reals which defines a euclidean space. Thus the conflation of concrete and abstract exchange-values leads smoothly to the unsupported conclusion that a "price space" is a euclidean space. [Debreu 1986, p. 1261]

It is on the basis of this presumed "fit of the mathematical form to the economic content" [Debreu 1986, p. 1259] that the whole neoclassical edifice, not just general equilibrium theory, has been constructed. At every point it presumes -- through the convenience of its conflation -- that a system of exchange or market values has the same structural properties, i.e., euclidean, as do the numbers that represent them. But this subconscious presumption, the most fundamental *hypothesis* of neoclassicalism, is easily tested when the conflation between concrete and abstract quantities is avoided.

6. Test Procedure

"How many snowballs would be required to heat an oven?" [Duhem, 1905, p.112] This joke, credited to Diderot, illustrates three verities of quantitative science: profound structural differences exist between various quantitative orders; their structures may diverge radically from that of ordinary arithmetic; and, most importantly, the structures of empirical quantitative orders are autonomous vis à vis human will and imagination.

In a more positive vein, but to a similar purpose, Bertrand Russell identified the principle by which science applies mathematics to empirical phenomena. "Whenever two sets of terms have mutual relations of the same type, the same form of deduction will apply to both." [Russell, 1937, p. 7] Application of arithmetical addition to mass, length and time are familiar examples. Yet, in such cases, where one set of terms is logical or mathematical and the other set is not, the existence of a homomorphism between the two sets is, as Diderot's jest illustrates, a purely empirical matter. It presumes the discovery of a set of extra-mathematical relations which repeated testing, not a set of axioms, shows to be structurally analogous to the arithmetical ones of =, <, > and +.

Monetarized markets assign to each set of commodities exchanged a denominate number describing a property--the "exchange-value" or "market-value"--of that set. It is, therefore, although unorthodox, eminently sensible to examine those exchanges as measurement operations. But the rarity of this approach and its false resemblance to demand theory's axiomatic method, makes it wise to begin by outlining and illustrating, even at the risk of laboring the obvious, the general empirical principles involved. For this purpose I will consider in some detail the procedures, and the logic behind them, for measurement of mass.

A loaded scales in level balance defines operationally a symmetrical, transitive and reflexive relation $=_m$ between a pair of masses. This physical relation, $=_m$, is an archetypal *empirical* equivalence relation. Similarly, a scales out of level balance defines the asymmetrical, transitive, and irreflexive precedence relations lighter than $<_m$ and heavier than $>_m$. I am particularly concerned with the property, or absence of, additivity, and with the fact that physics attributes this property to mass, not on the basis of any axiom or axiom set, but rather on the basis of the results of empirical operations. It is only after equivalence and precedence relations and a numerical scale have been developed and tested for a domain of objects that it becomes possible to *test* for additivity. This property, it must be emphasized, never holds absolutely for elements of a measurement order. Instead, additivity holds *only* relative to some particular mode of combination. Lengths, for example, are additive only when combined end-to-end in a straight line perpendicular to the axis of gravity. With the measurement of mass, the combining operation is merely the joint positioning of two objects in a pan of a scales and weighing them as a single object. In general, where $\bigcirc$ denotes an empirical combining operation and *p* the measurement number assigned to the object in the parenthesis, the additivity condition may be stated:

$$p(a \bigcirc b) = p(a) + p(b).$$

For mass, this condition is interpreted as follows. One puts object *a* on one of the pans of a scales which then is brought into balance with standard weights whose mass number m is observed. Object a is then replaced with object *b* and its mass is noted. Then the two objects are placed together on the scale, this being the combining operation for mass $\oplus$, and the measurement number of the combined mass is determined. From experience, one predicts that the mass number of *a* and *b* combined will be the arithmetical sum of the mass numbers of *a* and *b* weighed separately, i.e.,

$$m(a \oplus b) = m(a) + m(b).$$

With respect to the equivalence and precedence relations $=_m$, $<_m$ and $>_m$, and the operation of combining masses in the pans of a scales, the physical order of mass satisfies not only the additivity condition, but also a set of conditions structurally analogous to those satisfied by arithmetical addition in the set of positive real numbers $\underline{R}^+$. *Arithmetical addition* **+** together with the arithmetical relations =, < and > satisfy the following conditions for any elements x, y, z of $\underline{R}^+$.

A1. $x + (y + z) = (x + y) + z$ (associativity).
A2. $x + y = y + x$ (commutativity).
A3. $z + x = z + y$ implies $x = y$ (cancellation property of + with respect to =).
A4. $(z + x) < (z + y)$ implies $x < y$ (cancellation property of + with respect to <).
A5. $(x + y) > x$ (monoticity).
A6. For any x and y there exists an integer n such that $nx > y$ (Archimedean property).

Where $=_p$, $<_p$ and $>_p$ denote unspecified empirical equivalence and precedence relations for a property P, $\bigcirc$ an unspecified empirical combining operation, and a,b,c are elements of the set S of all objects possessed of the property P, then the empirical conditions corresponding to A1-A6 are as follows.

P1. $a \bigcirc (b \bigcirc c) =_p (a \bigcirc b) \bigcirc c$
P2. $a \bigcirc b =_p b \bigcirc a$.
P3. $c \bigcirc a =_p c \bigcirc b$ implies $a =_p b$
P4. $c \bigcirc a <_p c \bigcirc b$ implies $a <_p b$
P5. $a \bigcirc b >_p a$.
P6. For any a such that $a =_p a_1 =_p a_2 =_p \ldots =_p a_n$ and any b there is a nth power $a \bigcirc a \bigcirc \ldots \bigcirc a$ of a, written na, such that $na >_p b$.

It is important to note that the conditions P1-P6 refer to relations holding between empirical objects, not between measurement numbers describing those objects. But where an empirical order such as mass has been metricated--that is, where an easily reproducible object or process possessing the property has been chosen as a standard for defining a unit of that property--conditions corresponding to P1-P6 can be expressed in terms of measurement numbers and *arithmetical*, rather than empirical, relations. The P' list that follows includes as *P'3* the basic *additivity condition* noted above. Thus, where **p** denotes the measurement number assigned to the object in the parenthesis, we have:

P'1. $p(a \bigcirc (b \bigcirc c) = p(a \bigcirc b) \bigcirc c$
P'2. $p(a \bigcirc b) = p(b \bigcirc a)$
P'3. $p(a \bigcirc b) = p(a) + p(b)$
P'4. $p(c \bigcirc a) = p(c \bigcirc b)$ implies $p(a) =p(b)$
P'5. $p(c \bigcirc a) < p(c \bigcirc b)$ implies $p(a) <p(b)$

P'6. p(a ○ b) > p(a)

P'7. For any a such that $p(a) = p(a_1) = p(a_2) = \ldots = p(a_n)$ and any b there is a nth power a ○ a ○ . . . ○ a of a, written na, such that p(na) > p(b).

We will use these P' conditions to guide the inquiry into the structural characteristics of exchange-value. Each P' condition generates a question to ask about exchange-value: Does exchange-value satisfy condition P'x, and if not what corresponding condition does it satisfy? To answer these questions requires that various aspects of exchange-value noted in the introduction are examined *from the structural point of view*. These include market exchange as a measurement operation, downward sloping demand curves, inelastic demand, exchange-value units and, eventually, monetary inflation. In this way the investigation reveals a set of structural properties of exchange-value paralleling the P' conditions. This method of proceeding also has the advantage of placing the concrete structural properties of exchange-value in direct comparison to a structure, P'1-P'7, made commonplace through length and weight measurement. First the operational basis of exchange-value measurement must be explained.

7. Exchange-value as a Measurement Phenomenon

Everyday usage fails to differentiate between quantities of *things* and quantities of *measurement*. In science, however, the distinction between these two kinds of denominate numbers is fundamental. By definition, numbers of count have reference only to particular classes of objects. This property severely limits the scientific significance of count numbers, even when their units are based on measurable properties, for example, a pound of butter. Instead, it is the quantification of *properties* common to many classes of objects (for example, mass, length, time and exchange-value) which enables the generation of quantitative laws and of theoretical structures which transcend Aristotelian categories.

Just as mass and extension are properties common to all physical objects, so exchange-value is a property of all economic objects. It is on this basis that exchange-value must be judged a measurement order. But it is an extremely curious one, because the process of exchange-value measurement, that is, market exchange, is an integral and central part of economics' object of inquiry. Nevertheless, exchange-value, like the fundamental measurement orders of classical physics, is operationally defined in terms of a combining operation, a measurement object, an equivalence relation and a pair of precedence relations. Each of these elements of exchange-value measurement needs examination.

In a time period T, various categories of goods are exchanged for money in markets. In each of these markets, the placement in period T of multiple units of a good for exchange constitutes the empirical combining operation ⊙ for the measurement of exchange-value, just as the simultaneous placement of multiple objects on a scales for weighing defines the combining operation ⊕ for the measurement of mass. Physical objects combined under ⊕ for weighing constitute the "object mass"; similarly, the units of a commodity X combined under ⊙ constitute X's *market exchange or measurement set* for period T.

The market conditions of equilibrium and disequilibrium serve as equivalence and precedence relations for the measurement of exchange-value. A market which clears and is in equilibrium defines an exchange-value equivalence relation $=_e$ such that the exchange-value of the quantity of a good exchanged equals the exchange-value of the quantity of money exchanged for it. This relation parallels the equivalence relation used in weighing. When a loading of a scales results in a level equilibrium, the object mass is said to be equal to the combined mass of the mass standards ("weights") placed in the opposite pan. Markets which do not clear, like scales which do not balance, also identify quantitative relations. Excess supply or excess demand in a market defines an exchange-value precedent relation $<_e$ or $>_e$ such that the exchange-value of the commodity's market exchange set is less than or greater than the exchange-value of the money for which it was traded.

Still another parallel exists between mass and exchange-value measurement. In market trading, as in weighing, equilibrium is reached through trial and error. The initial price set for a commodity may or may not result in a market equilibrium, just as the initial number of units of standard mass placed in one pan may or may not result in an equilibrium of the scales.

As with the measurement of mass, the conditions of associativity and commutativity appear to hold for exchange-value by definition. Thus, where $\underline{e}$ denotes the exchange-value number, i.e., market-value, assigned to the homogeneous commodity set in parentheses:

E1. $e(a \odot (b \odot c)) = e((a \odot b) \odot c)$.

E2. $e(a \odot b) = e(b \odot a)$.

8. The Law of Demand as a Law of Negative Non-additivity

Despite the structural similarities between the measuring of mass and the measuring of exchange-value, the numerical results of the two sets of operations are structurally dissimilar. This disparity becomes obvious once the negative slope of commodity demand curves is interpreted in terms of the additivity condition. If exchange-values were additive relative to their measurement combining operation, then all demand curves would be horizontal. A demand curve's slope means that as additional units of the commodity are placed on the market for exchange, $\odot$, their exchange-value is not additive. Moreover, the fact that a demand curve slopes downward means that exchange-value is negatively non-additive in the following sense. On the basis of experiential knowledge of market exchange, one can predict: for a given demand, that the exchange-value of homogeneous commodity sets $\underline{a}$ and $\underline{b}$ combined into one market exchange set will be less than the arithmetical sum of the exchange-values of $\underline{a}$ and $\underline{b}$ if they are exchanged separately in different periods.
Thus, where $\underline{e}$ denotes the exchange-value number assigned to the market exchange set of commodity X in the parenthesis:

E3. $e(a \odot b) < e(a) + e(b)$.

9. Inelastic Demand

This section continues with the deployment of the properties P'1-P'7, which describe the structure of metricated empirical orders analogous to arithmetical addition in $\underline{R}^+$, as heuristic devices for identifying the structural properties of exchange-value. I wish to discover how the phenomenon of inelastic demand relates to the properties

P'4. $p(c \bigcirc a) = p(c \bigcirc b)$ implies $p(a) = p(b)$

P'5. $p(c \bigcirc a) < p(c \bigcirc b)$ implies $p(a) < p(b)$

P'6. $p(a \bigcirc b) > p(a)$.

The previous section showed how the inverse relation between price and quantity demanded means that, over the operation of market exchange, exchange-value is non-additive. Inelastic demand identifies a situation in which exchange-value is not only non-additive, but "negatively" so, in the sense that increasing the size of the market exchange set *decreases* its exchange-value. This condition generates relations which violate conditions P'4, P'5 and P'6.

I begin by considering P'6, because the inconsistency of inelastic demand with this property is self-evident. Interpreted in terms of exchange-value, inelasticity of demand means that beyond a certain size, the larger a commodity's market exchange set, the less its market-value and vice versa. Formally, when the demand for a commodity is inelastic or unitary and the sets in parenthesis are market exchange sets of the commodity, we have:

$$e(a \odot b) \leq e(a),$$

which is contrary to P'6. On the other hand, when the demand for the commodity is elastic:

$$e(a \odot b) > e(a).$$

Therefore the following relation characterizes exchange-value:

E6. $e(a \odot b) >, =$ or $< e(a)$.

Inelastic demand's inconsistency with conditions P'4 and P'5 vis à vis exchange-value and the operation of market exchange is only slightly less obvious. It is illustrated, in Diagrams One and Two on the following page, by a curve which I will call a "total exchange-value curve". It plots changes in a commodity's market exchange set's size to changes in that set's exchange-value. In other words, the total exchange-value curve shows how, as the number of units of a commodity traded in a period of time changes, the market value of that total changes.

In Diagrams One and Two, a, b, and c stand for quantities (or sets) of the market's commodity, whereas the quantities in parentheses stand for *market exchange sets*, that is the total quantity of the commodity exchanged in period T. Diagram One shows that the market exchange sets of ($c \odot a$) and ($c \odot b$) have the same exchange-value, but that the exchange-value of the market exchange set (a) is less than the exchange-value of that of (b). This violates condition P'4. Instead of P'4, the following relation characterizes, exchange-value:

E4. $e(c \odot a) = e(c \odot b)$ does not imply $e(a) = e(b)$.

Diagram Two shows that the market-value of the market exchange set of ($c \odot a$) is less than that of ($c \odot b$), despite the fact that the exchange-value of (a) is greater than that of (b). This violates condition P'5. Instead of P'5, the following relation characterizes exchange-value:

E5. $e(c \odot a) < e(c \odot b)$ does not imply $e(a) < e(b)$.

Thus, the existence of inelastic demand means that the conditions P'4, P'5 and P'6 do not hold for exchange-value.

10. The Commensurability Problem with Units of Account

A ratio scale representation of a quantitatively structured property P, such as mass or exchange-value, is constructed with a unit of the property defined on the basis of some object or process characterized by P and called the standard. [Campbell, 1928; Hempel, 1952; Feather, 1959; Carnap, 1966; Ellis, 1966; Krantz, 1971] But given the creation of such a scale, there still remains the empirical question of that scale's range of magnitude relative to that of the property to whose measurement it is applied. Existence of a ratio scale representation system across a property's full range of magnitudes, a "complete" as opposed to only a "partial" scale, means it is possible in principle to *combine* enough copies of the standard to make it P-commensurable, $=_p$, with every object in the domain. [Hempel, pp. 62-8] This corresponds to a weak version of the Archimedean conditions A6 and P6 and P'7 of Section 2.

When working with real models, economics concerns itself tangentially with the process of constructing a system of representation, including a scale, for exchange-value. Without money, it is necessary to adopt a commodity as a numeraire; and, regarding such designations, Pigou [1917] summed up what has remained the traditional wisdom regarding exchange-value's satisfaction of an Archimedean condition. In "The Value of Money" Pigou declared, "the value of any combination of commodities in general can be cited in terms of any single commodity." This "Pigou Hypothesis" may be stated formally for the aggregate endowment Γ as follows, where "a" is the standard unit of a commodity *A* and $\geq_e$ means equal or greater than in exchange-value.

For any *A* and any Γ there is potentially a nth power of a ⊙ a ⊙ ... ⊙ a of a, written na, such that na $\geq_e$ Γ.

Further thought, however, shows that not only is Pigou's hypothesis false, but also that there is no single commodity and only one combination of commodities in terms of which the value of the aggregate endowment can be cited. [Fullbrook, 1992] The argument by which this result is reached is disconcertingly simple. Just as a physical object only has weight in a gravitational field, so too a good only has exchange-value in the context of a system of markets, with its "forces" of supply and demand. If a good's exchange-value were equal to the exchange-value of the aggregate endowment, then it would be the only good in the endowment, in which case there would be no demand, no exchanges, no market and no exchange-value. A similar contradiction arises when it is assumed that the equilibrium exchange-value of a quantity of a good is more than half the value of the aggregate endowment. This structural attribute of exchange-value may be expressed formally as follows, where, as above, Γ indicates the aggregate endowment, "a" the standard unit of a commodity *A*, and $\geq_e$ means equal or greater than in exchange-value:

E7. For any Γ there is no *A* such that there is a nth power of a ⊙ a ⊙ ... ⊙ a of a, written na, such that the exchange-value of na is more than half the exchange-value of Γ. A corollary of E7 is that for any commodity there must be a quantity beyond which its demand ceases to be elastic, or, from this paper's point of view, beyond which the exchange-value of the commodity's market exchange set does not increase.

This obstacle to a full representation of exchange-value in a real economy can be overcome if and only if the aggregate endowment Γ is adopted as the standard of exchange-value. [Fullbrook, 1992] Under such a system, exchange-values would be represented as proportionate *parts of the whole exchange-value*, in the same way that probabilities are conceived and expressed as fractional parts of a certainty or of the universal event. These peculiarities of representing exchange-value in a real economy are further indications of exchange-value's idiosyncratic structure.

11. Monetary Inflation

Monetary inflation may be commonplace, but as a *measurement* phenomenon it is eccentric. [Fullbrook, 1992, 1993] Consider mass. Increasing the number of standard weights used in weighing operations does not decrease the mass of those weights. Likewise, increasing the number of standard rods used in measuring length does not change the length of any of those rods. But increasing the quantity of money exchanged, that is, the number of standards of exchange-value used in measuring the exchange-value of the component sets of the aggregate endowment--not only decreases the exchange-value of existing money tokens, it also decreases each one's value by the same proportion. This shows that, in still another way, exchange-value has a structure radically different from mass, length and arithmetical addition.

Consider further the case of mass. The property of mass is understood as a function of micromasses, whose existences are independent of the larger mass with which they are grouped. A body's mass is the totality of the masses of that body's parts, and its mass will increase if more parts are added to it. With quantitative properties of this type, each magnitude is *the aggregate of its parts*, the direction of determination running from the micro to the macro level.

But quantitative properties are not always of this type. Probability provides a relevant example. Certainty not only defines an upper bound for magnitudes of probability, but also serves as a whole in relation to which the probabilities of events in the probability space are conceived as parts. In other words, certainty, or the certain event, provides a *unique* standard of measurement for probability, with all other probabilities in the space being defined as parts of that "whole" probability. In this sense, just as the existence of a mass is based on a sum of its parts relation, an event's probability is based on a *part of a whole* relation.

As a means of gaining insight into the structure of exchange-value and its relation to token money, consider the case of an uniform probability space, such as drawing from a deck of cards. Under the concept of probability, each card in the deck shares equally in the "whole" probability, that is, the certainty that when a card from a deck is drawn one of the *n* cards in the deck will be drawn. Changing the number of cards in the deck changes each card's probability of being drawn, but does not change the probability of the certain event. Furthermore, despite the increase in the number of elementary events, and the change in every card's probability of being drawn, the sum of their probabilities remains equal to the probability of the certain event, and each card's probability remains equal to every other card's probability. A formally identical set of relations holds for the exchange-value of money. It is generally agreed that an increase or decrease in the number of units of money exchanged for the aggregate endowment Γ, although altering the exchange-value of every unit of money, and thereby changing the scale on which exchange-values are represented, does not change the exchange-value of Γ, nor disturb the equality between the exchange-value of MV and the exchange-value of Γ, nor eliminate the exchange-value equivalency between all the individual money tokens. These relations are loaded with structural significance, and so need to be added to the list of exchange-value's structural properties. Where m is a unit or standard token of money, n is the numeric of MV (i.e., nm = MV), Γ is the aggregate endowment, each of whose elements is exchanged only once, and all markets clear, the following relations describe operational aspects of exchange-value measurement.

E8. The exchange-value of Γ is constant for all magnitudes of MV.

E9. For any (numerical) value of MV, $MV =_e \Gamma$.

E10. Where nm = MV, for any value of n, $m =_e MV/n$.

We need to consider further these relations.

Token money affords a unique analytical opportunity, because with it the usual order of dependency between use-value and exchange-value is reversed. Token money has use-value only because it has exchange-value.[3] This reversal provides a case where the exchange-values of a homogenous set of objects are determined independently of multifarious use-value considerations. For this reason, token money offers a much clearer view of the structure of exchange-value than the one had from looking at the exchange-value of other commodities.

Of special interest are the properties of the relations by which the market system causes definite magnitudes of exchange-value to become associated with (i.e., become a property of) the individual money tokens. Most of all, it is important to note the direction of determination. Is the exchange-value of MV determined by the number and the exchange-value of the individual money tokens composing MV, that is, a micro explanation, or is the exchange-value of the individual tokens determined by the number of parts into which the exchange-value of MV is divided, that is, a macro explanation? If the market exchange process assigns an exchange-value directly to the standard money unit, then the exchange-value of MV would be *directly* proportional to MV. But if the primary assignment of exchange-value is to the total quantity of money exchanged, whatever its magnitude, then the exchange-value of the money unit would be *inversely* proportional to MV. The existence of inflation, as described above, shows that it is the latter process which characterizes exchange-value and money. The exchange-value of the total quantity of money exchanged equals that of the goods for which it exchanges, or, in an economy where all markets clear, the exchange-value of the aggregate endowment of goods (E9). This means that it is the aggregate endowment that acts as *the primary standard of exchange-value* (E8), with the exchange-value of each of the individual token-money units being determined as *equal parts* of the aggregate endowment's exchange-value (E10).

This inverse relation between the quantity of money and the exchange-value of its units corresponds to the negative non-additive law derived earlier with respect to the exchange-value of commodities (E3). But here, with the exchange-value of money, is a very special or pure case of this law, pure in the sense that it is characterized by a fully specified and invariant function--the rarest of all phenomena in the social sciences. Other things remaining unchanged, a percentage change x in the quantity of money exchanged decreases the exchange-value of a unit of money by 1/1+x. In other words, for a given Γ, a money unit's elasticity of exchange-value with respect to quantity of money exchanged is unitary, that is, a constant.

This section has used the empirical phenomenon of inflation to reveal the structure of the economic process by which token money comes to have the property of exchange-value. If, as with most quantitative orders, the determination of magnitude proceeded from the part to the whole, then an increase in the quantity of money exchanged would not only increase the exchange-value of MV, but would also leave the exchange-value of the individual tokens unchanged. In such a world, monetary inflation would be impossible. Instead, other things being equal, increasing MV affects the exchange-value of individual tokens in the same way as increasing the number of elementary events in an equiprobable probability space affects each of those events's probability.

12. Summary of Structural Properties of Exchange-Value Discovered

This paper's results thus far spring from two simple but novel analytical manoeuvres: raising the question "What is the structure of exchange-value?" and undertaking an analysis of the market system as a measurement system. This dual strategy affords a fresh analytical point of view on a much observed and often analyzed set of phenomena. Old verities and celebrated results may be scrutinized and further examined within the context of this new but complementary analytical framework. Thus, the law of demand (in the empirical sense of an observed regularity), is now interpreted as a law of negative non-additivity of exchange-value. Likewise, from relations expressed in the equation of exchange, a fully specified law of negative non-additivity of exchange-value has been derived for money. Because of the failure to look at price and quantity phenomena from the measurement and structural angles, these non-additivity laws, despite their simplicity, have not previously entered into economic thought.

The measurement/structural approach also raised questions about the representation of exchange-value, questions whose answers have not been backed previously by analysis. In considering this set of questions, it has been found that the commensurability condition in a real economy is satisfiable only if exchange-values are represented as fractional parts of the whole exchange-value, that is, the exchange-value of the aggregate endowment. Furthermore, it also is the aggregate endowment's exchange-value which serves as the primary standard of exchange-value in a monetary economy, with the exchange-value of the individual money units depending on *the number of parts into which the whole exchange-value is divided.*

There follows a list of the structural properties of exchange-value as a measurement order discovered thus far.

E1. $e(a \odot (b \odot c)) = e((a \odot b) \odot c)$.

E2. $e(a \odot b) = e(b \odot a)$.

E3. $e(a \odot b) < e(a) + e(b)$.

E4. $e(c \odot a) = e(c \odot b)$ does not imply $e(a) = e(b)$.

E5. $e(c \odot a) < e(c \odot b)$ does not imply $e(a) < e(b)$.

E6. $e(a \odot b)$ >, = or < $e(a)$.

E7. For any Γ there is no A such that there is a nth power of $a \odot a \odot ... \odot a$ of a, written na, such that the exchange-value of na is more than half the exchange-value of Γ.

E8. The exchange-value of Γ is constant for all magnitudes of MV.

E9. For any (numerical) value of MV, $MV =_e \Gamma$.

E10. Where nm = MV, for any value of n, $m =_e MV/n$.

13. Exchange-Value as a Boolean Algebra

The analysis of sections 7 and 8 entail certain scientifically curious requirements and possibilities for the numerical representation of exchange-value. Expressions E8, E9 and E10 show that the exchange-value of money, the operational standard, is related to the primary standard in an idiosyncratic but highly structured way. Under the terms of the convention of monetary exchange, MV's exchange-value is identically equal to Γ's, meaning that changes in MV change the exchange-value of MV's units and not the exchange-value of MV itself. Under these relations, the exchange-values of the units of money are *equal parts of the given whole exchange-value*, that is, the exchange-value of Γ. Analysis also has shown that the commensurability condition can be satisfied for exchange-value metrication, be it a monetary or a real economy, only by using Γ, the aggregate endowment, as the primary measurement standard. This is a situation completely at odds with the physical measurement orders of mass, length and time, where, vis à vis the commensurability condition, there exist an infinite number of possible primary standards for metrication. Since, for these orders, the choice of standard is ultimately arbitrary, so too is the size of the unit. Therefore, these orders can be metricated at best on a ratio scale, meaning that their measurement numbers are unique only up to a similarity transformation. But exchange-value, since for a given aggregate endowment there is only one possible primary standard, may, like probability, be metricated on an absolute scale, that is, represented as fractional parts of the exchange-value of its singular standard. Such exchange-value numbers would be unique up to an identity transformation. The fact that exchange-value invariably has been represented on a ratio scale is irrelevant to the present inquiry. What is important is that analysis has shown that in principle those ratio scale representations can be converted to absolute ones by dividing them by MV.

Quantities of money and quantities of commodities exchanged can be interpreted as *money exchange sets and commodity exchange sets*.[2] The special sets corresponding to MV and the set of all units of all commodities exchanged in period T will be termed the *money exchange space* or Γ_M and the *commodity exchange space* or *Γ*, respectively. This yields $P(M_\Gamma)$ and $P(\Gamma)$, the power sets of M_Γ and Γ, which, each being a set of all subsets of a given set, are boolean algebras. Because, as shown above by E8, E9 and E10, the exchange-value of M_Γ is always uniquely determined by Γ such that $M_\Gamma =_e \Gamma$, the exchange-value of money can be uniquely represented as parts of *the whole exchange-value*, i.e. as parts of the exchange-value of Γ. Furthermore, since all of the elements (units of money) comprising M_Γ are interexchangeable, i.e. of equal exchange-value, M_Γ is a *uniform* or *equivaluable* exchange space. It follows that where,

$$M_\Gamma = \{m_1, m_2, \ldots, m_n\}$$

and a money exchange set A is formed of g money units, that e(A), the exchange-value of A, is g/n parts of the whole exchange-value $e(\Gamma)$. For every commodity exchange set, monetary exchange also identifies a money exchange set of equal exchange-value. Thus, when MV or the cardinal of M_Γ is known, it is possible to satisfy the following conditions:

D1. With every element A of $P(M_\Gamma)$ and every element A of $P(\Gamma)$, it is possible to associate a non-negative number $e'(A)$, the *absolute exchange-value* of A, such that $0 \leq e'(A) \leq 1$.

D2. $e'(M_\Gamma) = e(\Gamma) = 1$.

D3. If A and B are mutually exclusive subsets of M_Γ or of Γ, i.e., if $A \cap B = \varnothing$, then $e'(A \cup B) = e'(A) + e'(B)$.

D4. For every $e'(A)$, where A is a subset of M_Γ or of Γ, there is an element $e'(A')$ such that $e'(A) + e'(A') = e'(M_\Gamma) = e'(\Gamma) = 1$.

Condition D1 specifies a set $E_{M\Gamma}$ or E_Γ of absolute exchange-value numbers for $P(M_\Gamma)$ or for $P(\Gamma)$ respectively. *Conditions D1-4 determine that $E_{M\Gamma}$ and E_Γ are boolean algebras*. (See Appendix for the axioms of boolean algebra.)

Under the boolean hypothesis, increasing or decreasing the size of a commodity's exchange set assumes a theoretical significance which *transcends the individual commodity market*. Any change in the size of a single market exchange set means that the commodity exchange-space Γ also changes. If Γ_1 was the original exchange space and A_1 was the ath commodity's market exchange set, then, after $A_1 \odot A_2$, the new exchange space is $\Gamma_2 = \Gamma_1 \cup A_2$. *This represents a change in the standard of exchange-value.* Consequently, the exchange-values of the elements of $P(\Gamma_1)$ and $P(\Gamma_2)$ are not *strictly* comparable. The algebraic analysis of value identifies every exchange-value as a part of a whole exchange-value, and $e(A_1)$ and $e(A_1 \odot A_2)$ represent exchange-values which are parts of two different wholes. This attribute of being defined only relative to a particular universal set, which is the ultimate basis of the index number problem, is a peculiarity of the boolean structure. This relativity property manifests itself in other boolean fields. For example, for every set of statements there is a different statement calculus, and the probability numbers of events belonging to different event spaces are not combinable by arithmetical addition.

14. Applications

This section considers the relevance of the foregoing results to a pair of theoretical problems which have resisted solution. Both anomalies appeared when analysis began from a non-zero degree of aggregation.

a. The homogeneity problem

After the appearance of *The General Theory* (1936), efforts were made to overcome the micro/macro split that had always been the fault-line of economic analysis. These attempts centred on the integration of money into the general equilibrium model. But these efforts at synthesis revealed an antinomy that was potentially as undermining to existing economic theory as the results of the Michelson-Morley experiment had been to classical physics. The impasse centred on the so-called "homogeneity postulate". Whereas the n equations for excess demands in the commodity markets were homogeneous of degree zero in money prices, the one equation for the excess demand for money was homogeneous of degree one in money prices and the quantity of money, an apparent contradiction where the excess demand for money is taken to be identically equal to an excess supply of commodities. [Patinkin, 1956; Fullbrook, 1992, 1993, 1994]

Seen through the lens of this paper's structural analysis, the homogeneity difference in the monetarized general equilibrium model, far from being a problem, is precisely what is predicted. This difference arises, according to the new approach, because the money demand equation introduces a quantitative order with a different structure from that of the ratio scale of money prices. In other words, the monetarized model of GET brings directly into play the boolean structure of exchange-value. Exactly how this introduction takes place needs examination.

When the *exchange-values* of quantities of money are considered at the micro level, i.e., the exchange-values of subsets of M_Γ, they exhibit the same structural properties as the n quantitative orders of the n commodities. In other words, if M_1, M_2, and M_3 are disjoint subsets of M_Γ--as, for example, three ten-dollar bills in one's wallet--then their exchange-values are combinable according to the conditions A1-5, as are also the probabilities of independent events belonging to the same event space. But when the exchange-value of money is considered at the *macro* level, as is the case when a change in M_Γ is entertained, then the analysis is brought up against exchange-value's boolean structure, especially its universal set which acts as an upper bound, thereby imposing the degree-one homogeneity on the demand for units of money.

Instead of a structural crack to be papered over, the apparent contradiction thrown up by the attempt to integrate money into the general equilibrium paradigm may be viewed as the thin end of a heuristic wedge. Further research, one suspects, will reveal that exchange-value's boolean structure is integrally tied up with the micro/macro "paradoxes" revealed by Keynes, and which, half a century on, even after the brilliant analyses of Clower and Leijonhufvud, remain largely intellectually intractable.

b. The Fisher - Becker Curio

Gary Becker [1962, 1971] showed that from the existence of budget constraints alone it follows that a *market* demand curve must slope downward, whether consumer behavior is rational or not. By beginning his analysis at the level of the market, rather than at the level of the individual, Becker shows that there exists a macro budgetary effect which entails "the basic demand relations". [Becker, 1971, p. 11] In other words, scarcity of funds is a condition for negative sloping market demand curves, and rational consumer behavior is not a necessary condition. In fact, Becker showed that the whole of the subjective side of demand theory is otiose when it comes to the deduction of the general inverse relation between price and quantity demanded at the market level.

Becker's result needed an explanation which would integrate it into a larger theoretical edifice, but none was forthcoming. Without this theoretical grounding, Becker's "scarcity principle" remained an anomaly and, therefore, survives today only as a curious and obscure footnote to economics, with students continuing to be taught that the downward slope of the market demand curve is due to intrasubjective factors.

But Gary Becker was not the first to point out the budget effect, that is, "the effect of a change in prices on the distribution of opportunities." [Becker, 1962, p. 6]. With a different emphasis and at a higher level of aggregation, Irving Fisher noticed and expounded on the same macro dimension of *relative* prices forty years earlier. In *The Purchasing Power of Money*, Fisher writes:

> if one commodity rises in price (without any change in the quantity of it or of other things bought and sold, and without any change in the volume of circulating medium or in the velocity of circulation), then other commodities must *fall* in price. The increased money expended for this commodity will be taken from other purchases. [1920, p. 178]

Note that here Fisher is not considering the absolute level of prices, which he assumes constant, but rather relative prices and how at the macro level there exists an interdependency between them. Fisher realized, in a way that Becker did not, that his discovery posed a major paradox for economics. [1920. p. 180] Furthermore, because Fisher carries the analysis to a higher level of aggregation, he comes much closer than does Becker to discovering the boolean structure of exchange-value. His nearness to this elementary truth becomes apparent when two terms of his foregoing passage are translated into the terminology of the present paper. Substituting "exchange-value per unit" for Fisher's "price" and "measured exchange-value" for his "money expended" yields the followings statement:

> If one commodity rises in exchange-value per unit (without any change in the quantity of it or of other things bought and sold, and without any change in the volume of circulating medium or in the velocity of circulation), then other commodities must **fall** in exchange-value per unit. The increased measured exchange-value for this commodity will be taken from [the exchange-value of] other purchases.

Thus translated, Fisher's passage captures "the part of the whole" relation which, as for probability, is one dimension of exchange-value. It captures also the fact that, despite all the concrete individuality that goes into exchange-value's making, the exchange-value of every unit of every commodity is irreducibly and fundamentally a SOCIAL relation that extends as far as does the economy in which the exchange of the unit takes place.

What Fisher could not do was explain how the micro and macro levels of "causation" were linked. His insight, like Mendel's discoveries concerning heredity, lacked a contemporaneous means of explanation. Abstract algebra was little known, and without this set of tools *the boolean structure* which links exchange-value's two levels of "causation" could not be identified.[4]

15. Conclusion

Quantitative orders possess structures. This paper has investigated the structure of exchange-value and found that it is an unsuitable subject for ideological or metaphysical purists. Market forms of economic value are reducible neither to wholes nor to relations between atoms. Because of its boolean structure, to increase the probability of one event decreases the probability of another and vice versa. Likewise for exchange-value. Every exchange-value in a money economy exists only as an integral and interdependent *part* of an intersubjective system of other exchange-values. For entrenched intellectual positions, these results have obvious negative implications. But in closing I prefer to emphasize their possible positive importance for understanding two significant current problems.

One is the need to bring ecological considerations into economic decision-making. Disclosure of the boolean structure of exchange-value means that attempts to intellectually come to grips with the problem by placing money-values on ecosystems and other mega ("trans-boolean") entities are, like basing engineering calculations on speeds in excess of the speed of light, radically misplaced and counter-productive. Many ecological economists have suspected as much, but found it difficult to argue against the euclidean commonsense of neoclassicalists.

The second problem is the redistribution of income and wealth from the poor and middle-classes to the rich and super-rich now taking place both intra- and internationally at a rate and on a scale unprecedented in human history. No adequate theory exists to explain and thereby to enable us to curtail, stop or reverse this radical change in the human condition. Of course it has something to do with globalization. But why should globalization have this redistributive effect? And how can the process be managed so that human will controls the direction and magnitude of the redistribution? This paper provides a theoretical framework in which to think about the problem.

Globalization means that all monetary exchanges are pulled into one giant system of exchange-value. Nothing much may have changed nationally, but the market-value of commodity X no longer exists primarily in relation to the other commodities of that country. Instead each person's economic assets are valued by *a different system* than before, by a global system that brings new forces into play.

The boolean structure of exchange-value means that the neoclassical dichotomy between value and distribution theory is spurious. For a century economists have taught themselves and others to think of value and distribution separately. This doctrine stands in the way of understanding globalization. The present analysis has shown that ultimately value and its distribution are, as earlier thinkers intuited, merely different ways of looking at the same thing. To explain one is to explain the other. To explain why any commodity or any person's day of work is worth so many dollars or euros or yen is to explain why the world's income and wealth are distributed the way they are.

Appendix: Axions of a Boolean Algebra

A boolean algebra is a 6-tuple [B,∪,∩,′,0,1], where B is a set, ∪ and ∩ are binary operations in B, ′ is a binary relation in B having B as its domain, 0 and 1 are distinct elements of B, and for all a, b, and c belonging to B the following axioms are satisfied.

1. Each operation is associative:
   $a \cup (b \cup c) = (a \cup b) \cup c$ and $a \cap (b \cap c) = (a \cap b) \cap c$.
2. Each operation is commutative:
   $a \cup b = b \cup a$ and $a \cap b = b \cap a$.
3. Each operation distributes over the other:
   $a \cup (b \cap c) = (a \cup b) \cap (a \cup c)$ and $a \cap (b \cup c) = (a \cap b) \cup (a \cap c)$.
4. For all a in B,
   $a \cup 0 = a$ and $a \cap 1 = a$.
5. For each a in B there exists a ′-related element a′ such that
   $a \cup a' = 1$ and $a \cap a' = 0$.

Endnotes

1. Likewise, Arrow and Debreu's famous proof of the existence of an equilibrium in a competitive economy proceeds on the basis of the assumption that the system of prices is an euclidean space. [Arrow and Debreu, 1983]

2. For a very accessible account of these fundamentals see Carnap 1966, pp. 51-124.

3. The demand for token money is a pure case of the type of market situation analyzed by the French Intersubjectivists. See Dupuy, 1989; Levy, 1991, 1994; Orléan, 1990, 1992; and most especially 1998. See also Fullbrook 1994b, 1996b.

4. This boolean structure, with its internal relations, is consistent with Hodgson’s reconstitutive downward causation, where “it is impossible to take the parts as given and then explain the whole,” because the whole to some extent shapes its parts. [See Hodgson’s essay in this volume. (p. 14)]

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DIAGRAM 1

revenue or
exchange-value

Market Total Exchange-value Curve
for Commodity X

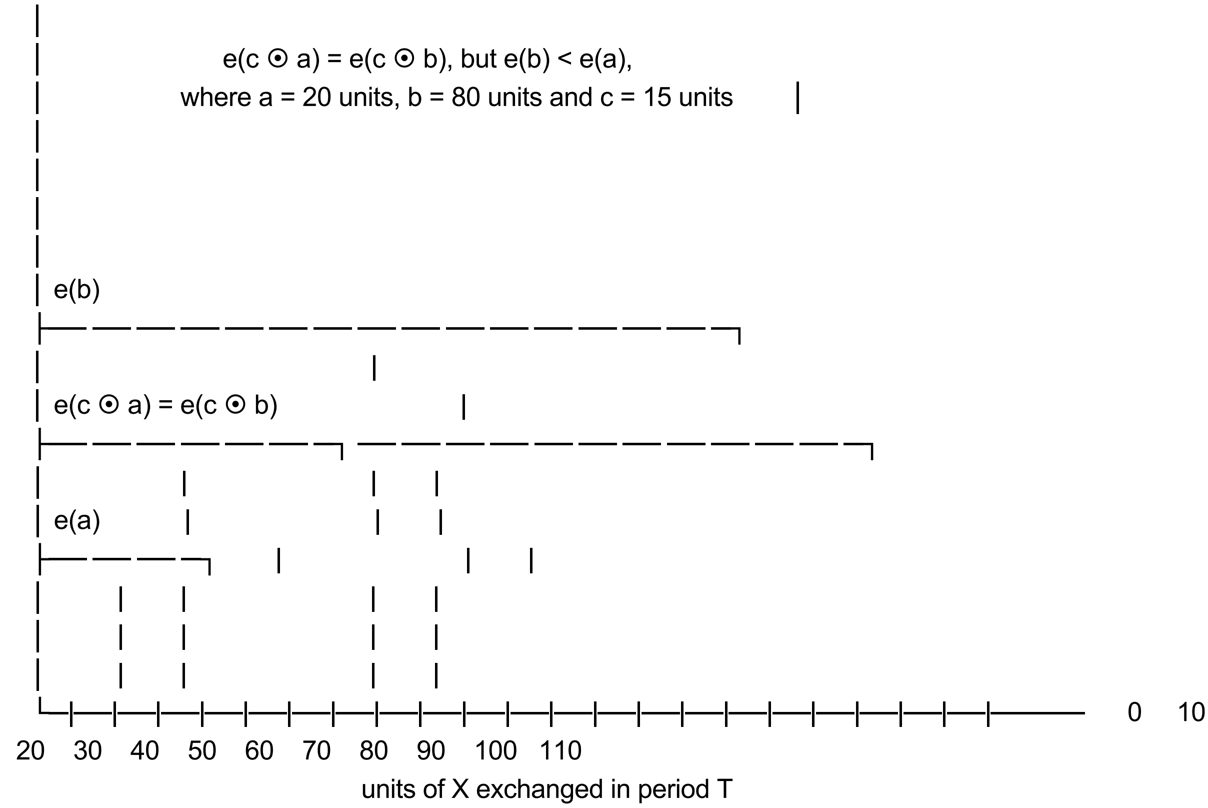

DIAGRAM 2

revenue or
exchange-value

Market Total Exchange-value Curve
for Commodity Y