real-world economics review, issue no. 95
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On the relevance of the Church-Turing Thesis for theoretical economics

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Abstract
Nearly a century ago, Alonzo Church and Emil Turing famously proposed that any calculation performed in the actual world by any physical means (e.g., pencil and paper) could also be executed on a computer. Since its inception, the Church-Turing Thesis (CTT) has been extensively debated, especially in computer science, and the philosophy of mind. Recently, in economics, CCT has been referenced (Gräbner et al., 2019) as a rationale for a new approach in method and theory. Based on CTT, it is possible to convert the equations of mathematical economics into computer simulations. The conversion makes possible the identification of important variables concealed by equations. Discovery is also enhanced by the exploratory manipulation of simulation components. Together these changes furnish the basis for a refined formalism. The method may thus be described as a “dialog of models”. The present article-essay describes this approach, and a pioneering application by Albin and Foley (1992) to the Arrow-Debreu model (1954) of general equilibrium. It is concluded that the approach should be broadly applied to the Neoclassical Framework.

Key words: Computational economics, Agent-based models, Equation-based models, Arrow-Debreu model

JEL codes: C02 C63 E13

1. Introduction

The emergence of Computational Economics – itself but a single dimension of the broader revolution in Information Technology (IT) – is poised to dramatically shape the future development of economic theory. We are not primarily talking about the justly celebrated advances in processing speed. For example, we do not understate the importance of new architectures that combine logic and memory functions thus reducing the time required for information to pass between them – an obstacle traditionally known as the “von Neumann bottleneck” (Peper, 2017). But here we are looking at something which, although related to speed, is different: a kind of computational “dialog” between an empirically-based, economic model and its mathematical expression – an exploratory dialog for which the outcome cannot be predicted, regardless of how precisely the initial conditions are specified (Gräbner et al., 2019). This semi-autonomous interplay between simulation and formalism is anchored in the Church-Turing Thesis (CTT) (Church, 1935). The thesis, informally, states that any computation carried out in the “real world” (e.g., on pencil and paper) can also be carried out on a computer. (A somewhat more formal version will be given in Section 2). In this brief essay we hope to suggest that CTT may have significant implications for the development of economic theory.

The essay begins with an informal overview of CTT emphasizing “real-world” and machine computation. This perspective is then applied to agent-based models (ABMs) and equation-based models (EBMs), two methods extensively used in the social and natural sciences. In this discussion we hope to show that the current controversy regarding ABMs “versus” EBMs amounts to a false dichotomy and that, in the light of CTT, the two strategies may be viewed as strongly complementary (Gräbner et al., 2019). These correlated approaches are then applied to economic theory. As an example of what is possible, Peter Albin and Duncan

Foley's (1992) coupled-ABM-EBM analysis of the Arrow-Debreu (AD) model is presented in some detail. Finally, we point out that the strategy outlined here, although valuable, should be seen as a subset of a potentially larger platform in which simulation and formalism interact and enrich one another.

2. The Church-Turing Thesis: logic and computing machines

The closely linked theoretical constructs developed by Alonzo Church and Emil Turing in the 1930s known as the Church-Turing Thesis (CTT) centered around the notion of computation. This concept was understood in an abstract generalized sense as a physical or "mechanical" process in which specific input to some system was subjected to a set of actions or operations that, in the absence of error, yielded the corresponding output (Copeland, 1997). In Church's original version of the thesis – it was never a formal proof, but more closely resembled a "working hypothesis" (Post, 1936) – an idealized human agent working tirelessly with some physical instrument (perhaps pencil and paper) correctly solved a mathematical function. Against this backdrop, Church proposed a logical structure known as the Lambda Calculus – today it is recognized as an early programming language – which, given the same input values as the mathematical drudge, could duplicate its output (Copeland and Shagrir, 2019). In essence, Church metaphorically posited that any mathematical function which could be successfully executed in the actual world could also be successfully executed through the use of his lambda calculus.

This was when Alan Turing made his mighty entrance. In an argument that famously defined an idealized computing machine, Turing claimed that the latter was computationally equivalent to Church's lambda calculus. More precisely, Turing (1936) imagined a device which serially scanned either a 0 or a 1 from a moving, one-dimensional tape; each successive scan determined the configuration of the machine. (For a valuable discussion and illustration, see Penrose, 1989, pp. 30-73). Thus, in this reformulation, the logical operations performed on a numerical input in Church's lambda calculus became machine operations determined by a binary code. Put differently, Church had claimed that any "real-world" computation (symbolized by his minimalist workers) could be executed through lambda calculus. Turing had followed with the claim that any computation executed through lambda calculus could be executed through his Platonic machine. The conclusion was straightforward: any computation in the actual world can be carried out by a Turing Machine (TM). This is the variant of CTT most widely understood today (Copeland and Shagrir, 2019). In the remainder of this essay, we examine the relation of Church and Turing's remarkable thesis to simulation, formalism and the development of economic theory.

3. Simulations, equations, and the Church-Turing Thesis

Turing's 1936 claim of computational equivalence has cast a long shadow, occasionally assuming a sweeping, metaphysical cast. Much of the latter discussion has centered on the real-world computations (Wolfrain, 2002, pp. 637-714) which, if only in principle, could be executed on a TM. The claim seemed clear enough in relation to Turing's – and Church's – abstract human workers, perhaps equipped with pencil and paper (Copeland and Shagrir, 2019). But what of other entities? Are enzyme interactions a computation (Zauner et al., 2001)? What of electrostatic interactions in the neural membrane (Price and Wallace, 2003)? The debate is valuable because it serves to remind us that CTT is evolving, a critique originally voiced by Emil Post (1936) who viewed CTT as a “working hypotheses” (as noted above).

A dimension of this discussion is the relationship between computational simulation and mathematical representation, or formalism. As a case in point, we may consider Agent-based models (ABMs) and Equation-based models (EBMs) typifying simulation and formalism, respectively. An ABM is an artificial micro-world comprised of heterogeneous, autonomous units (agents) which formulate decisions (outputs) based on decisions of other agents (inputs) (Bruch and Atwell, 2015). ABMs are frequently multi-level. An agent may be, for example, an individual, a political action committee, or even a corporation. This fine-grained structure contrasts with the (typically) more concise mathematical expression of the EBM (Gräbner et al., 2019). A mathematical object – most often a differential equation or set of such equations – an EBM describes, and can be used to predict, the change in state of a complex system. This method has been extensively used in engineering and is increasingly applied in medicine (Daun et al, 2008).

Importantly, the platforms are the subject of a growing debate: which platform is better? ABMs, their proponents note, can capture a richness of detail that is often concealed by equations (Gräbner et. al., 2019). Moreover, they are a generative strategy which can reveal emergent properties unforeseen by the investigator (Epstein, 1999). EBM modelers counter that the use of differential equations lends exactitude and predictive power to the descriptions of dynamic systems (Daun et al, 2008). These exchanges, although illuminating the advantages of each method, have all too often been presented as a dichotomy, overlooking their complementarity and the value of a synthetic approach.

CTT supplies the rationale for a correlated strategy. Any ABM, as a TM program, can convert the equations of an EBM into a set of machine instructions that yield a qualitative representation. Moreover, the similarity of ABMs and EBMs, which promotes their combined application, has sometimes been understated in the critical literature. Seeking to correct this oversight, Richiardi (2018) has emphasized that an ABM is grounded in a fully mathematical system. The functions underlying an ABM relate to input-output transforms: “They are logical theorems saying that, given the environment and the rules described by the model, outputs necessarily follow from inputs” (Richiardi, 2018, p. 33).

4. Simulations, equations, and neoclassical economics

Can the combined, exploratory application of simulation and equation modeling, consistent with CTT, significantly contribute to theoretical economics? The state of the art is contentious. For well over a century, the science has been guided by a set of idealized axioms, configured as differential equations formulated by Léon Walras (Turk, 2012). At the center of the Neoclassical viewpoint and the attached controversy is General Equilibrium Theory (GET). Walras envisioned a perfectly competitive economy, comprised of rational actors, in which supply matched demand, thereby yielding a stable state (Walras, 1969 [1874]). Based on the perspective developed above, it is not our intention to provide yet another critique of GET, nor defend its continued application. (For a generally sympathetic recent discussion of GET, see Köllmann, 2008; for a severely critical view see Ackerman, 2002). Instead, we hope to show that the combined application of ABM and EBM approaches to GET in particular, and the Neoclassical Framework in general, may yield unexpected insights into economic theory. As a case in point, we may consider Albin and Foley's 1992 computational study of the Arrow-Debreu (AD) model (1954).

The roots of AD extend to the beginnings of Classical economics. Adam Smith (1776) devoted extensive discussions to multiple markets in which the price of one commodity affected the price of another (e.g., wheat and potatoes) in a system typically displaying a dynamic stable state, i.e. a general equilibrium (Maskin, 2019). A century later, Léon Walras (1874) attempted a more elegant version, expressed through an auction metaphor: An auctioneer coordinated a multiple-commodity market, matching producers with consumers, arriving at a set of market-clearing prices. But the model was problematic. Each commodity transaction was described by a distinctive equation; an economy of M commodities would require M equations (Maskin, 2019).

This limitation was addressed by Kenneth Arrow and Gérard Debreu (1954). The AD model was the elegant mathematical construct that Walras had sought but had not achieved. The EBM assumed strategically more complex producers and consumers. Producers were described by inputs, outputs, and a plan (or "production set") which included, most significantly, the maximization of profit. Consumers were described by endowments, and a plan ("consumption set") which included preferences within budgetary constraints. These eminently rational agents transacted within a game-theoretic economy, coordinated by an auctioneer who sought maximization of demand. Based on these and several other assumptions, the AD model proved (to the authors' satisfaction) that general equilibrium was possible in multiple markets.

The AD model of GET has given rise to over three generations of supportive and negative commentary (e.g., Köllmann, 2008; Ackerman, 2002). This history suggests the fallacy of the excluded middle. Perhaps a more valuable stance is to neither bury AD nor to praise it. Rather, we should view it as a methodological portal to expanded investigations. This approach was adopted by Albin and Foley (1992) in a simulation study which examined the effect on AD if certain key assumptions were changed. Consistent with CTT, the formalism of Arrow and Debreu was modelled as an ABM. The auctioneer was eliminated, yielding a decentralized economy. The new model also assumed more fluid interactions among producers and consumers. The revised AD model:

> "showed how the absence of the Walrasian auctioneer leads to an increasing inequality as a consequence of decentralized trading... In particular, it showed the impact of decentralized trading and the effect of different network structures underlying the trading relations of the agents" (Gräbner et al., 2019, p. 765).

This finding would appear consistent with recent research indicating a causal relationship between homogenous social networks and economic inequality (Jackson, 2019).

5. Conclusion

The broad claim by Alonzo Church and Emil Turing that any mundane computation by any mechanical means could be generated by a computer may have significant implications for economic thought. CTT could serve as a foundational principle for the structured interaction of equation and simulation to derive novel perspectives from an established theoretical framework. Seen in this light, the equilibrium demonstration of Kenneth Arrow and Gérard Debreux is neither "the high altar of mathematical economics" (Durlauf, 2017) nor a defunct formalism "still dead after all these years" (Ackerman, 2002, p.19). Rather, in the midst of contention, there is a third approach. Neither mainstream nor pluralist, the present method may generate unanticipated theoretical insights. The latter, in turn, may inspire a revisionist mathematics of subtlety, realism, and increased predictive power.

We may venture a step farther. The proposed discovery procedure, in addition to reciprocally enriching simulation and formalism, may reveal unexpected affinities of competing theoretical views. Such, indeed, may be the deeper significance of Albin and Foley's research. If a modified version of AD resembles network economics, perhaps the differences between Neoclassical theory and alternative perspectives has been exaggerated. And perhaps, accordingly, we should envision a research tradition – unbiased, empirically-grounded, linking equation and simulation – which may yield challenging approaches by bridging conceptual divides.

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SUGGESTED CITATION:
Wallace, Ron (2021) "On the relevance of the Church-Turing Thesis for theoretical economics." *real-world economics review*, issue no. 95, 22 March, pp. 124-129, http://www.paecon.net/PAEReview/issue95/Wallace95.pdf.

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