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A diagrammatic derivation of involuntary unemployment from Keynesian micro-foundations

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Abstract
This paper lays the micro-foundations for Keynesian macroeconomics. It shows that there is only one geometrical demand curve which does not assume that aggregate demand is constant. This curve may therefore be regarded as the Keynesian demand curve. Further, it shows that this curve aggregates without distortion. This means that the macro-economy can be constructed from heterogeneous agents without invoking a representative agent. Finally, it shows how the aggregate labour demand curve interacts with the labour supply curve to yield involuntary unemployment.

JEL Classifications E12, E13

Keywords Keynes, macroeconomics, micro-foundations, involuntary unemployment, representative agent

Introduction

One of the principal charges against the economics of Keynes is that it lacks micro-foundations. This charge is usually hurled by the New Classical school, whose own micro-foundations consist of a single representative agent, which, many would say, amounts to no micro-foundations at all.

Keynesians themselves have, however, by and large, not attempted to build micro-foundations for the Keynesian macroeconomic superstructure. The argument generally advanced is that it is unnecessary or impossible or both.

In the conclusion to a previous paper (George, 2016) I suggested that General Equilibrium theory and Marshallian analysis are mathematically equivalent and that this had implications for Keynes's position on involuntary unemployment. In this paper I not only construct micro-foundations for Keynesian macroeconomics but show that they negate one of the principal assertions of New Classical economics, viz, that there is no such thing as involuntary unemployment.

Keynes' claim in *The General Theory* was that classical economics dealt with the special case in which aggregate demand was constant, and that his own book dealt with the more general case in which no such constraint was placed on aggregate demand. I show that linear and other demand curves implicitly assume that aggregate demand is constant, and that there is only one geometrical curve which does not make this assumption. That curve is the rectangular hyperbola. Since it meets Keynes' criterion of generality, it may be considered as the Keynesian Demand Curve.

Linear demand curves have the additional feature that they are distorted under aggregation. The Keynesian demand curve, on the other hand, scales up without distortion, maintaining its shape from the level of the individual or firm right up to the aggregate economy. With the

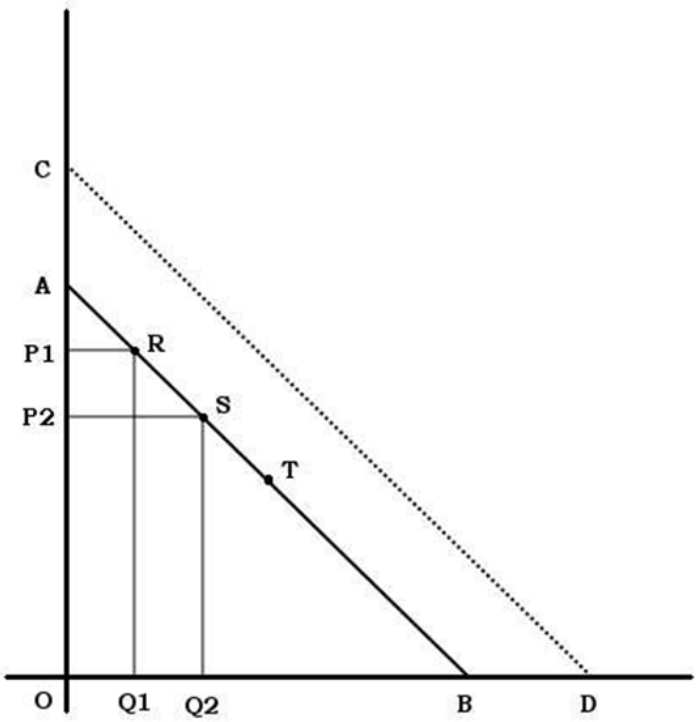
Keynesian demand curve it is therefore possible to construct the macro-economy by aggregating heterogeneous individuals and firms in the economy. My analysis thus calls into question the concept of the representative agent, which constitutes the micro-foundations of New Classical Economics.

Finally, I show how the aggregate labour demand curve, constituted by adding up the labour demand curves of all the firms in the economy, interacts with the labour supply curve to yield involuntary unemployment, a result quite at variance with other schools of macroeconomics.

A. The demand curve

The demand curve is one of the building blocks of economics. In textbooks the demand curve is sometimes depicted as a line, sometimes as a curve, sometimes both. The assumption is that the actual shape of the curve does not matter so long as the slope is negative throughout. The figure below shows the market's demand curve for fish as a line.

Figure 1 Linear demand curves for fish



The original demand curve is AB. The demand curve is drawn assuming that people's incomes are constant as are their tastes. If people's incomes increase then at every price they can buy more fish and the demand curve moves to CD. Similarly, if people develop an increased taste for fish the demand curve moves to CD.

T is the midpoint of the demand curve. The segment AT is the elastic zone. The segment TB is the inelastic zone.

Assume that the initial equilibrium is at point R (in the elastic zone) where the price is P1 and the quantity sold is Q1. Assume also that at this point individuals spend all their income and

do not save anything. Next suppose that because of a movement of the supply curve the price falls to P2, also in the elastic zone, as a result of which the equilibrium moves to S. The quantity of fish bought increases to Q2. We can also see from the graph that the money spent on fish rises; the initial amount spent is the area of O-P1-R-Q1 and the final amount spent is the area of O-P2-S-Q2.

But here we run into a problem. We had assumed that individuals spent all their income at the first equilibrium point R and now we find them spending a larger amount on fish at S. This can happen only if they spend a smaller amount on some other good or goods so as to maintain their spending constant.

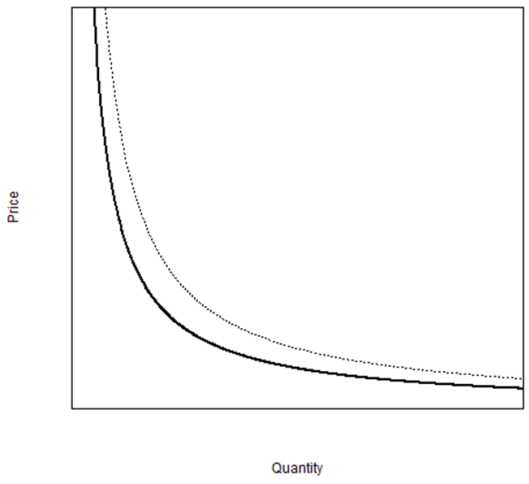
To summarize, along the elastic portion of a linear demand curve, when the price of fish falls not only does the quantity of fish bought increase but the money spent on fish also increases. So the money spent in other markets has to fall so as to maintain our initial assumption of a constant income. In general, the money spent at any point on each half of the linear demand curve is different from that spent at any other point. To compensate for this difference the money spent in other markets, and therefore the price and demand in those markets, has to change.

This property of linear demand curves is also shared by demand curves of other shapes, with a solitary exception: the rectangular hyperbola PQ = constant shown in Figure 2.

When the price of fish falls between two points on a rectangular hyperbola, the quantity of fish bought increases. But the money spent on fish at the two points remains the same since PQ is a constant. Though the quantity of fish bought rises the increase does not come at the expense of spending on any other good. Between two points on a linear demand curve spending is in general different. In a family of rectangular hyperbolas such changes of spending are effected by movements *between* demand curves, not *along* a demand curve.

The rectangular hyperbola is thus the only demand curve which does not assume that aggregate spending is constant. Keynes' claim for the *General Theory* was that unlike classical economics, it dealt with the general case in which aggregate spending was free to change. The rectangular hyperbola meets this criterion. It may thus be considered as the Keynesian Demand Curve.

Figure 2 Demand curves as rectangular hyperbolas



B. The addition of demand curves

Figure 3 shows two linear demand curves. To add two demand curves we note that prices cannot be added but quantities can. So for each price on the y-axis, we note the respective quantity for each demand curve and add them to get the quantity at that price on the aggregated demand curve. Figure 4 shows the demand curve formed as a result of the addition. The first point to note is that unlike the constituent demand curves the aggregate demand curve is discontinuous. If many linear demand curves of a variety of magnitudes are added the aggregated demand curve will be an assembly of disconnected line segments that get closer and closer to a curve. If every linear demand curve is identical to CD then the combination of n demand curves will be a line segment hinged at C. The intersection with the x-axis will be at a point whose distance from the origin is equal to n times OD

Figure 3 Two linear demand curves

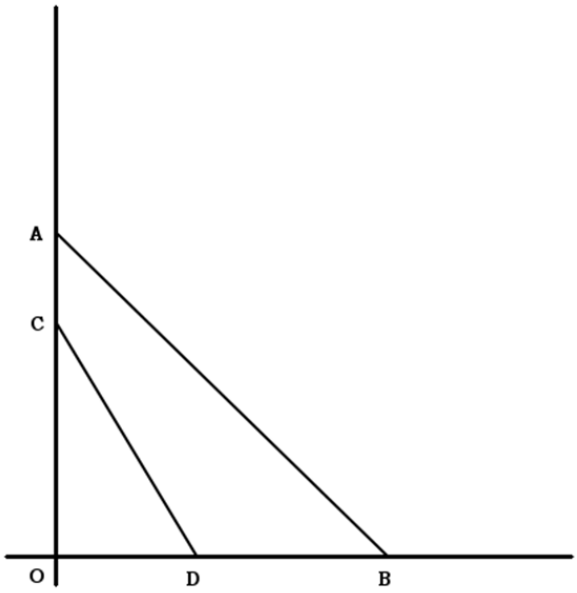
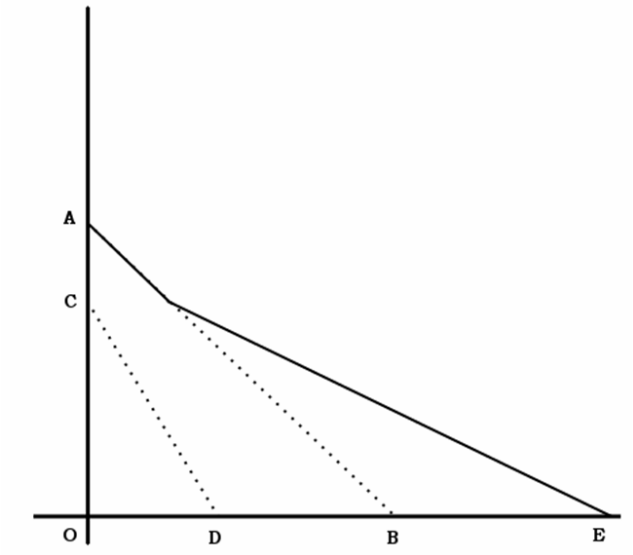


Figure 4 Addition of two linear demand curves



If the representative agent is taken as the smallest (or lowest) demand curve (which is a line) then the behaviour of the aggregate demand curve (which is a curve) will be completely different from the behaviour of the representative agent. Even if the representative agent is taken as the average demand curve the behaviour of the aggregate demand curve will be quite different. Only if the representative agent is a scaled down version of the aggregate demand curve can its behaviour be regarded as mirroring the aggregate demand curve. But in that case, it is not the aggregate demand curve which is being constructed from the representative agent but the representative agent that is being constructed from the aggregate demand curve. Or, put another way, we would be constructing the micro-foundations from the aggregate economy, not the other way round. If every linear demand curve in the economy were identical, then the aggregate demand curve would approach closer and closer to the horizontal; its behaviour would thus be completely different from that of the demand curves which constitute it.

The rectangular hyperbola, which has been previously introduced as the Keynesian demand curve, suffers from no such limitation. Rectangular hyperbolas when added yield rectangular hyperbolas. It is easy to prove this. Consider three rectangular hyperbolas PQ = a, PQ = b, and PQ = a + b. At price P1, the first rectangular hyperbola gives us Q1 = a/P1. At the same price, the second rectangular hyperbola gives us Q2 = b/P1. Adding the two gives us Q1 + Q2 = (a + b)/P1, which is of course Q3, the quantity at that price for the third rectangular hyperbola.

What this means is that no representative agent is required in order to construct the Keynesian macro-economy. Every individual's demand for fish is a rectangular hyperbola. The small consumer's demand curve will be a rectangular hyperbola close to the origin; the large consumer's demand curve will be a rectangular hyperbola farther from the origin. Adding every such demand curve will give us the market demand curve for fish, which will be a rectangular hyperbola. And adding the demand curves of every good and service, all of which are rectangular hyperbolas, will yield the aggregate demand curve, which will again have the shape of a rectangular hyperbola. The aggregate demand curve will be the farthest away from the origin.

From the arithmetic of addition used above, it should be clear that the rectangular hyperbola is not the only curve that aggregates without distortion. For example, QP^2 = constant or Q.logP = constant also aggregate to yield new curves of the same type. They do not, however, meet Keynes's criterion of generality because the product of P and Q varies from point to point along a demand curve.

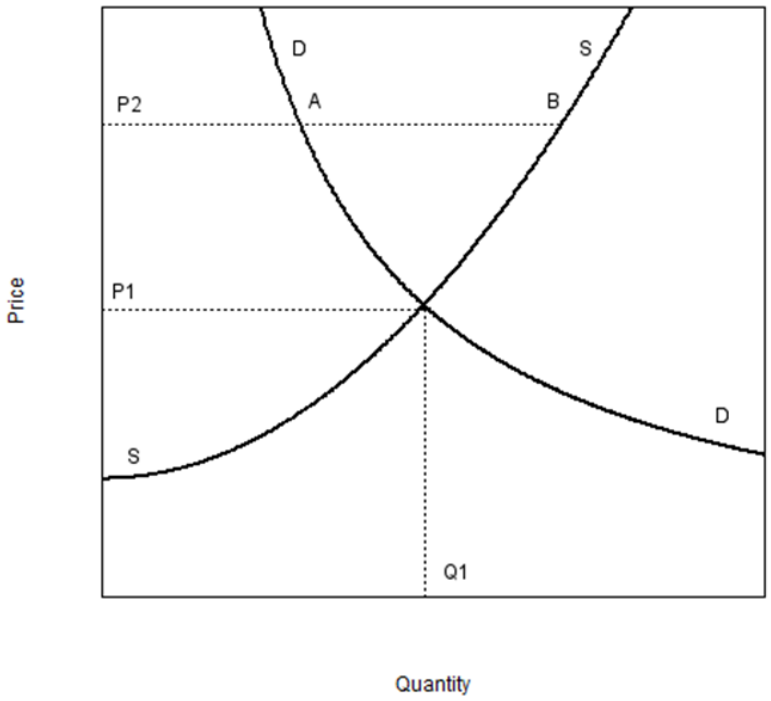
With the basic groundwork in place we now examine the classical view of unemployment.

C. The classical analysis of unemployment

Figure 5 shows demand and supply curves for labour, DD and SS respectively. The two intersect at a point where the wage is P1 and the quantity of labour demanded is Q1. This is an equilibrium point. If the wage rises to P2, the demand for labour falls and the supply of labour rises. More workers are willing to work at that wage than firms are willing to employ. The length of the segment AB is a measure of the unemployment generated at that wage. So workers reduce their wage demands until equilibrium is again reached at the original wage P1.

This is the classical analysis of the labour market. Unless a regulatory authority sets the minimum wage at P2 or trade unions set their wage demand at P2, which is higher than the equilibrium price, involuntary unemployment is impossible.

Figure 5 The wage floor

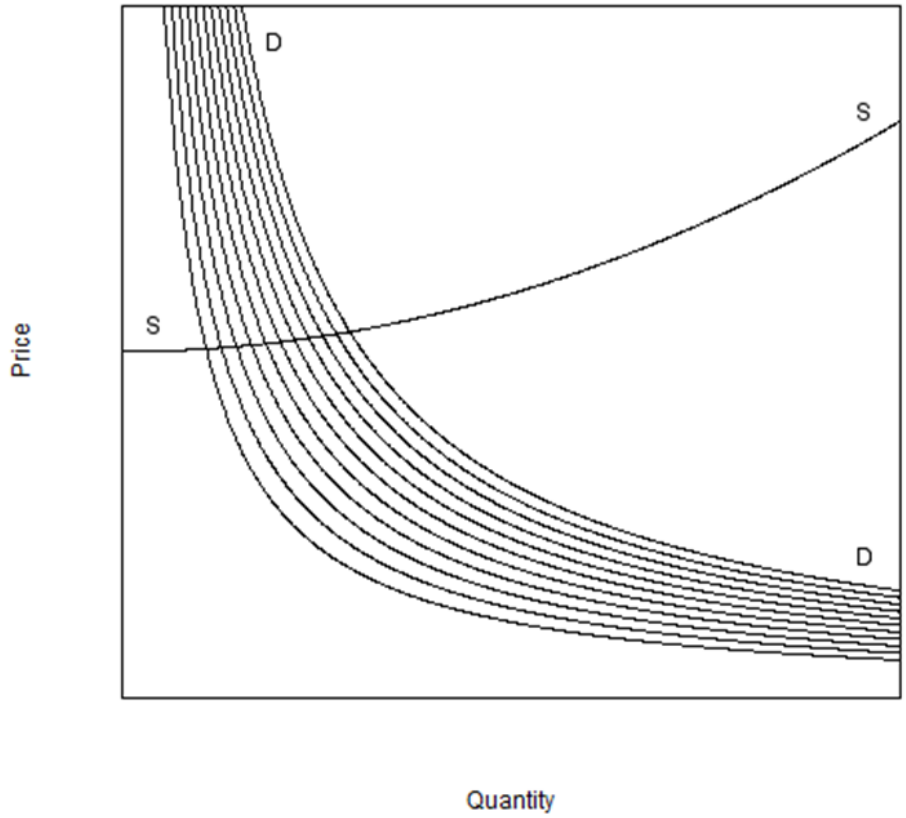


Most modern schools of economics generally accept this basic analysis. For instance, the New Keynesian school believes that sticky wages, not a wage floor per se, are the cause of unemployment. In the Stiglitz-Shapiro model (Shapiro & Stiglitz, 1984) too it is a wage above the equilibrium wage as in Figure 5 that is the cause of unemployment. In order to prevent workers from shirking firms pay them a wage that is higher than the market clearing wage. It is this differential that is the cause of unemployment during recessions. The New Classical school simply assumes that there is no such thing as involuntary unemployment; workers voluntarily choose to trade leisure for employment.

D. Involuntary unemployment resulting from the Keynesian demand curve

In section B we showed that every market in the economy from the smallest to the largest (aggregate demand) can be plotted on a single graph as rectangular hyperbolas. Small markets are located on demand curves close to the origin; large markets are located far from the origin. The labour market has one feature that sets it apart from other markets; it is by far the largest market in the economy. An idea of its size can be had from the fact that compensation paid to employees in the US amounts to about 44% of GDP at present (St Louis Federal Reserve, 2017). Labour is a necessary input in every good and service. This means that even if other markets shrink by small amounts, those small amounts are cumulative so that the demand curve for the labour market (which is a rectangular hyperbola) falls by the largest magnitude among all markets. Only the aggregate demand curve falls by a greater amount. Figure 6 depicts the fall and the interaction with a supply curve.

Figure 6 A labour supply curve cutting through a family of demand curves



It will be seen that at every consecutive intersection of the supply curve with a falling demand curve the part of the rectangular hyperbola at the intersection becomes closer and closer to the vertical.

But when the demand curve is vertical a change in wages (nominal or real) will not result in any change in employment. There is nothing that workers can do, individually or as a class, that will increase aggregate employment. The problem lies with the demand for labour and there is nothing that suppliers of labour can do to remedy the situation. By using a demand curve which does not assume that aggregate demand is constant (or, in other words, by using a Keynesian Demand Curve) we have thus arrived at a theoretical situation consistent with real-world involuntary unemployment.

Figure 6 is somewhat at variance with what Keynes himself felt about involuntary employment in *The General Theory*. In Chapter 2 he wrote:

> "Men are involuntarily unemployed if, in the event of a small rise in the price of wage-goods relatively to the money-wage, both the aggregate supply of labour willing to work for the current money-wage and the aggregate demand for it at that wage would be greater than the existing volume of employment."

There are two parts to this. In the second part Keynes argued that a fall in real wages will result in an increase in the aggregate demand for labour whereas Figure 6 suggests that when the demand curve is vertical a fall in wages will have no impact on employment demand. Three years later Keynes (Keynes, 1939) changed his mind, though with some

reservations, on the subject after studies showed that money wages and real wages tended to move in the same direction, not in opposite directions as Keynes believed. “That I was an easy victim of the traditional conclusion because it fitted my theory is the opposite of the truth,” Keynes wrote. “For my own theory this conclusion was inconvenient, since it had a tendency to offset the influence of the main forces which I was discussing and made it necessary for me to introduce qualifications, which I need not have troubled with if I could have adopted the contrary generalisation favoured by Foxwell, Mr Dunlop and Mr Tarshis.” He added in a footnote that Chapter 2 “is the portion of my book which most needs to be revised”.

If Keynes was reluctant to abandon the relationship between falling real wages and increasing employment demand it was because he believed in the correctness of what he called the first fundamental postulate of economics: “The wage is equal to the marginal product of labour.”
At first sight it indeed appears to be incontrovertible. Assume that the marginal product of labour is greater than the wage. In that case the capitalist can maximize his profit by hiring additional workers and raising output until the point that the marginal product of labour equals the wage.

Beyond that profit will fall. So the first postulate appears to be correct.

What Keynes does not seem to have realized is that his own theory disproved the first postulate. Remember that the aggregate demand curve is the only rectangular hyperbola that lies further out from the origin than the labour demand curve. During recessions it falls by an even larger magnitude than the labour demand curve. Therefore, the aggregate demand curve too is vertical at the point where it intersects the aggregate supply curve during recessions. Hence our argument that the capitalist can hire additional workers and increase output is incorrect; a vertical aggregate demand curve means that any additional output would remain unsold. The first fundamental postulate is therefore best regarded as an inequality: The wage is equal to or less than the marginal product. Note also that to disprove the first postulate all we need is the fact that aggregate demand has hit a wall. It does not matter whether the marginal product of labour is increasing or decreasing, a question which exercised Keynes a lot. We also note in passing that our argument casts doubt on the principle of profit maximization by firms.

Studies that show output and employment falling when wages are stickier than prices do not prove causation. When a capitalist finds his widgets remaining unsold (because aggregate demand is falling though he is not aware of this) he does not react by cutting wages; doing so will not help him get rid of his unsold inventory. He believes his widgets are not selling because he has overpriced them. So he first cuts prices and, perhaps, output. It is only when lower prices do not result in higher sales that he cuts jobs and wages in the hope of lowering prices further. The arrow of causation runs from falling aggregate demand to sticky wages and not the other way round.

The second part of Keynes’ definition of involuntary unemployment is interesting. It says that when there is involuntary unemployment the aggregate supply of labour will increase even when real wages are falling. The purpose of stating this appears to be to counter the argument that if unemployment is greater during a recession it has nothing to do with demand but is because workers choose not to supply their labour at the lower price or, in other words, voluntarily substitute leisure for work. This is contradicted by the fact that, as the aftermath of every recession shows, employment expands by large quantities even when real wages are

falling. See graph of median real earnings for workers, 16 years and over (St Louis Federal Reserve, 2017 (b)).

That employment rises for a long time after recessions without significant increases in the real wage suggests that the labour supply curve is horizontal during that period.

It is instructive to compare Figure 6 with Figure 5, which constitutes the classical proof that involuntary unemployment is impossible. Two differences are apparent. Figure 5 only considers movements along a demand curve whereas Figure 6 considers movements across demand curves. A classical demand curve for any good or service implicitly assumes constant aggregate demand, so it is only natural that it cannot result in involuntary unemployment, which is a feature of economies in recession. But even if Figure 5 considered a series of falling demand curves it would not result in "involuntary" unemployment if the demand curves were parallel to each other, which is how most textbook demand curves are drawn. For involuntary unemployment to occur it is necessary to have a family of curves that neither intersect nor are parallel so that the intersections of the supply and demand curves take place at points where the demand curves are closer and closer to the vertical.

We stated earlier that Keynesian demand curves are not the only curves that aggregate to give other curves of the same family. Figure 7 shows a family of curves of the kind Q.logP = constant.

Figure 7 Curves of the kind Q.logP = constant

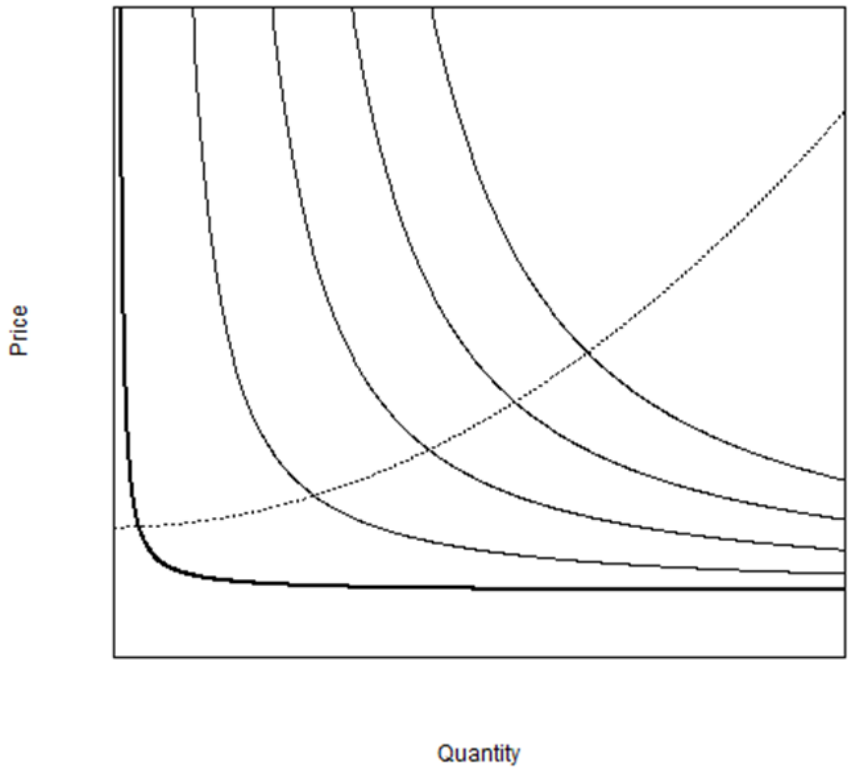
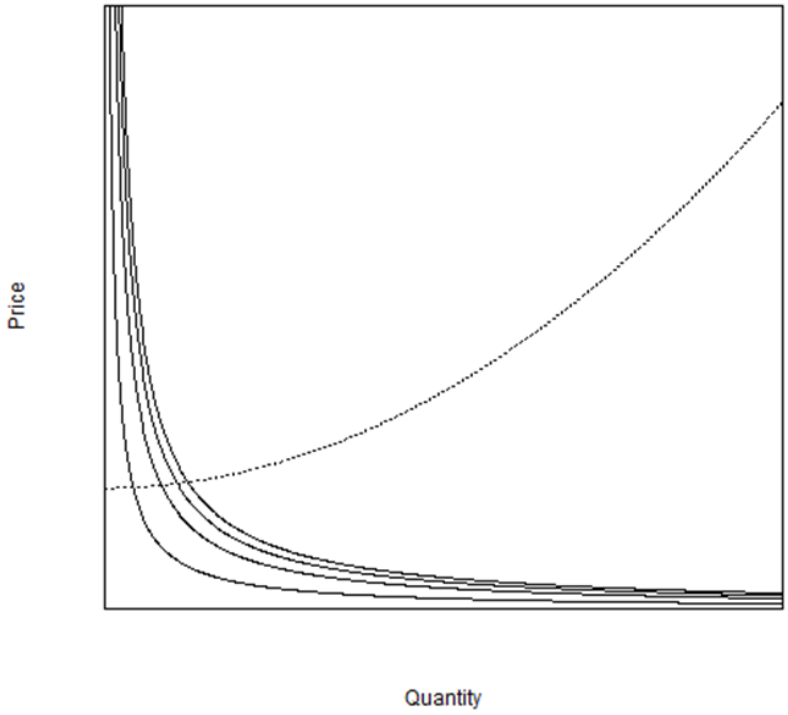


Figure 8 shows curves of the kind $Q.e^{P}$ = constant



Both families of curves on interaction with the labour supply curve show involuntary unemployment. Neither curve of course meets Keynes' criterion of generality. We depict them only to show that many curves that aggregate without distortion (and thus do not need the invention of a representative agent) result in involuntary unemployment. Demand curves that result in involuntary unemployment are quite common, not the impossibility that the New Classicals claim.

E. The minimum wage and elasticity of labour demand

Economic theory is clear that when the minimum wage is increased low-wage employment is negatively affected. Empirical evidence about the relationship is not, however, so unambiguous.

One review of a large number of studies (Neumark & Wascher, 2006) found that measured elasticities varied from well below -1 to well above zero although the authors concluded that the overall evidence supports the idea that increasing the minimum wage causes disemployment effects.

The problem with most of these studies lies in the metric used to measure the effect of a change in minimum wage on employment: the elasticity of demand for labour. It is defined as $\frac{\Delta Q/Q}{\Delta P/P}$ where P refers to the wage and Q to the quantum of employment. Since wages and employment move in opposite directions this is theoretically always negative. On the face of it using the elasticity of demand for labour to measure the effect of minimum wage changes on employment is plain common sense. The assumption is that if two studies in two separate locations show the same elasticity of demand for labour this means that employment is equally responsive to a change in the wage rate at both locations. Similarly, a higher elasticity

of demand, it is assumed, means a greater responsiveness of employment to a change in the minimum wage than a lower elasticity of demand.

But although this idea is universally accepted it is incorrect as can be seen from a look at the rectangular hyperbola. The elasticity of the rectangular hyperbola is equal to 1 everywhere. This suggests that at every point on the rectangular hyperbola, employment is equally responsive to a change in the wage rate. But we know that the rectangular hyperbola at points close to the origin along the x-axis is nearly vertical and that at points far from the origin along the x-axis it is nearly horizontal. In the nearly vertical portion a large change in the wage produces only a small change in employment. In the nearly horizontal portion of the rectangular hyperbola a small change in the wage produces a very large change in employment. So clearly elasticity is the wrong metric to measure the responsiveness of employment to a change in the wage rate. The correct measure to use is $\frac{\Delta Q}{\Delta P}$, which is the reciprocal of the slope.

In non-recessionary periods the labour supply curve cuts the demand curve at points where it is relatively horizontal. During recessions the labour supply curve cuts the demand curve at points where it is relatively vertical. The slope of the rectangular hyperbola (and thus its reciprocal) at the intersection with the supply curve traverses a range of values. This implies that the responsiveness of employment to a change in the minimum wage can vary widely, depending on when the study is performed, which is what the empirical data suggest. If studies show a wide range of effects of changes in the minimum wage it is not because some of those studies are badly designed but because reality is that way.

Interestingly, the graph says that in recessionary periods the minimum wage can be increased by a small amount without having a negative effect on employment and that in non-recessionary periods increasing the minimum wage can have deleterious effects on employment. This may appear to violate common sense. The explanation is that in non-recessionary periods, firms with very small profit margins can survive in the market. Increasing the minimum wage harms their margins and causes them to go out of business, thus contracting employment. During recessions the firms that survive are likely to have larger margins and are thus in a better position to weather an increase in the minimum wage.

Note, also, that changes in demand (measured in dollars) reveal themselves in movements of the rectangular hyperbola, not in movements along the curve. So large increases of the minimum wage may cause some firms to go out of business. This may result in the aggregate labour demand curve moving downward. But equally, small increases in the minimum wage can result in the demand curve moving up because of positive changes in income and spending and thus in aggregate demand.

Conclusion

In this paper we showed that the rectangular hyperbola is the only geometrical demand curve that meets the Keynesian condition of generality, i.e. it does not assume aggregate demand is constant. Moreover, it has the interesting property that it aggregates without distortion. Thus, it is possible to construct the Keynesian macro-economy by aggregating the demand curves of every agent in the economy, whether consumer or firm. A representative agent is therefore

rendered unnecessary. Finally, when this family of Keynesian demand curves for labour, which happens to be the largest market in the economy, interacts with the supply curve for labour involuntary unemployment follows as a matter of course. Most other goods and services constitute smaller markets, and in their case, the Keynesian analysis approximates to the Marshallian demand analysis.

Humans tend to view reality through the lens of theory. Before Copernicus it was evident to every child that the sun revolved round the earth. After the Copernican theory gained acceptance it has been apparent to every child that the earth revolves around the sun.

Before Keynes every economist was convinced that there was no such thing as involuntary unemployment. After Keynes, for some decades at least, there really seemed to be such a thing as involuntary unemployment. After the rise of New Classical Economics it was again obvious to most economists that involuntary unemployment did not exist. This paper, which demonstrates involuntary unemployment in theory, will, it is hoped, again change the way that economists view reality.

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