real-world economics review, issue no. 75
subscribe for free

A note on the aggregated production function and the accounting identity

Martin Zerner [Université Paris Diderot and Université Nice Sophia Antipolis, France]

Abstract
In this note, we give a more general, and drastically simpler, proof of the "Shaikh's relation" between the accounting identity and the Cobb-Douglas function, when the shares of the factors of production are constant. We also show that it is possible, with our simpler approach, to give a first order approximation of the error made when the shares of the factors vary slightly.
.

In a recent review of Jesus Felipe and McCombie's important and excellent book, *The Aggregate Production Function and the Measurement of Technical Change: Not Even wrong*[1], Bernard Guerrien and Ozgur Gun[2] remind us how decisive was Shaikh's criticism of Solow's famous paper "Technical Change and the Aggregate Production Function"[3]. Shaikh proves that Solow's, and others, "good results" – closely fitting data – can be explained by an accounting identity, completed by a "stylized fact" (the shares of the factors are almost constant in the data examined)[4].

The aim of our note is to give a more general, and drastically simpler, proof of the relation between the accounting identity and the Cobb-Douglas function, when the shares of the factors of production are constant. We also show that it is possible, with our simpler approach, to give an approximation of the error made when the shares of the factors vary slightly.

A purely algebraic proof of Shaikh's result

Shaikh's differential and integral reasoning is unquestionable as far as Solow's article is concerned. In a different setting it is somewhat strange to introduce a continuous parameter and to differentiate with respect to it, only to integrate back after a small rearrangement of terms.

It is much simpler to proceed directly, algebraically, in the following way.

Let the six real numbers *V, L, J, w, r* and *a* satisfy the two relations:

---

[1] Felipe and McCombie (2013)

[2] Guerrien and Gun (2015) , http://www.paecon.net/PAEReview/issue73/GuerrienGun73.pdf

[3] Solow (1957)

[4] Shaikh (1974)

(1) $V \equiv wL + rJ$ (accounting identity)

(2) $wL = aV$ (stylized fact).

The output, in value, V is equal to the payroll wL plus the interests served rJ, where J is the value of capital. The number a gives the part of "labour" in total revenue ($a = wL/V$). It follows from (1) and (2) that the part of "capital" rJ/V is $1 - a$.

Now, the term $(wL)^{b}(rJ)^{1-b}$, with any b, can be developed in two different ways:

$$(wL)^{b}(rJ)^{1-b} = w^{b}L^{b}\, r^{1-b}J^{1-b} \quad \text{(evident)}$$

and

$$(wL)^{b}(rJ)^{1-b} = (aV)^{b}\,[(1-a)V]^{1-b} \quad (\text{as } wL = aV \text{ and } rJ = (1-a)V)$$

$$= a^{b}(1-a)^{b-1}V$$

Equating the two forms of $(wL)^{b}(rJ)^{1-b}$, we obtain:

$$V = a^{-b}(1-a)^{b-1}w^{b}\, r^{1-b}\, L^{b}J^{1-b}$$

$$= A\, L^{b}J^{1-b}.$$

Then the relation:

(3) $$V = a^{-b}(1-a)^{b-1}w^{b}r^{1-b}L^{b}J^{1-b}$$

holds true for every real number b.

Two remarks before we go on.

Remark 1. This result is more general than Shaikh's, as it is valid for *every real number b* (and not only for the factor's share a). It can be deduced noting that for *any* homogeneous function $F(\cdot)$ of degree 1, we have:

$$F(wL, rJ) = F(aV, (1-a)V) = V \cdot F(a, (1-a)),$$

and then:

$$V = [F(a, (1-a))]^{-1} \cdot F(wL, rJ).$$

In Shaikh's case: $F(x_1, x_2) = x_1^{b}x_2^{1-b}$.

Remark 2 (geometrical proof). Consider V, L, and J as coordinates in a three dimensional space. The accounting identity $V \equiv wL + rJ$ restricts the data to a two dimensional plane. If the ratio wL/V is fixed, then they are further restricted to a half straight line from the origin (taking into account that all the numbers involved are positive). But on such a line, all homogeneous functions of degree 1 are equal to within a constant multiplicative factor.

An approximation of the error when the shares of the factors vary slightly

In order to get rid of the complications due to the factors w and r, we set:

$\pi = rJ$ (interest served) $W = wL$ (payroll) $V = \pi + W$ (product) .

Suppose we have a set of data (W_i, π_i) and we want to approximate them by a Cobb-Douglas function:

$$V^*_i = C\, W_i^b \pi_i^{1-b}.$$

Setting:

$$a_i = W_i/V_i,$$

we get from (3) :

$$V_i = B(a_i) W_i^b \pi_i^{1-b}$$

where

$$B(a) = a^{-b}(1-a)^{b-1}$$

So, given the relation (1), we choose

$$C = B(b),$$

and the relative error

$$(V_i - V^*_i)/V^*_i = (B(a_i) W_i^b \pi_i^{1-b} - C W_i^b \pi_i^{1-b})/C W_i^b \pi_i^{1-b}$$

is:

$$[B(a_i) - B(b)]/B(b)$$

.

Differentiating $B(\cdot)$, we get:

,

which implies the important result:

$$B'(b) = 0.$$

Consequently, given the Taylor expansion:

$$B(a_i) - B(b) = (a_i - b)B'(b) + (a_i - b)^2 B''(b)/2 + \ldots,$$

the relative error is of second order with respect to $a_i - b$.

It is easy to compute $B''(s)$ in order to find the principal part of the error. In the case of the American industry from 1909 to 1949, studied by Solow and Shaikh, the relative error never exceeds 1%.

Let us insist, as Shaikh does, that all *this is sheer mathematics* and does not involve any economic assumption.

It is a pleasure to thank Bernard Guerrien for his interest in this note and his help in editing it.

Bibliography

Felipe, J. and J. McCombie (2013), *The Aggregate Production Function and the Measurement of Technical Change*: *Not Even Wrong* (Edward Elgar)

Guerrien, B. and Gun, O. (2015), "Putting an end to the aggregate function of production... forever?", *Real-world Economics Review*, issue no. 73, 11 Dec 2015, pp.99-109,

Shaikh, A. (1974), "Laws of Production and Laws of Algebra: The Humbug Production Function", *The Review of Economics and Statistics*,56(1)

Solow R. (1957), "Technical Change and the Aggregated Production Function", *Review of Economics and Statistics*,39(3)

Author contact: zerner@paris7.jussieu.fr

SUGGESTED CITATION:

Martin Zerner, "A note on aggregate production function and accounting identity", *real-world economics review*, issue no. 75, 27 June 2016, pp. 152-155, http://www.paecon.net/PAEReview/issue75/Zerner75.pdf