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Demand theory is founded on errors

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1. The issue is applicability of mathematical operations

The applicability of the operations of algebra and calculus is a foundations-of-science problem. These operations have been applied incorrectly and where they are not applicable in microeconomic theory, the theory of games, decision theory and throughout the social sciences because the conditions for applicability of mathematical operations have not been identified in the literature. The applicability of these operations, in particular in demand theory, is founded on errors that are analyzed in detail in Barzilai [2 and 3].

In a recent paper [7], Katzner, whose work [6] contains these errors, says that:

> "Jonathan Barzilai, in a paper entitled, 'Inapplicable Operations on Ordinal, Cardinal, and Expected Utility' has raised important issues regarding ordinal utility, and correctly clarified the meaning of the general notion of ordinality in terms of the mathematical theory of measurement. In that process, he has also subjected the traditional theory of consumer demand to serious attack."

Having said that the meaning of the notion of ordinality has been correctly clarified, Katzner proceeds to obfuscate it by claiming a second notion of ordinal utility:

> "Barzilai's assault on traditional consumer theory, which is based on the mathematical theory of measurement, is useful because it brings to the fore the fact that, for economists, there is a second notion of ordinal utility, older than and independent of the mathematical-theory-of-measurement concept, and which is the relevant one for the traditional theory of consumer demand. That older approach seems to have had widespread acceptance among economists before the newer mathematical approach was known to them."

The following should be noted:

1. My analysis is not based on the mathematical theory of measurement. As Katzner notes, the title of my paper is "Inapplicable Operations on Ordinal, Cardinal, and Expected Utility." The subject of applicability of operations does not appear in measurement theory (see [7, 10, and 11]). Furthermore, in Section 3.8 of [2] I show that the mathematical theory of measurement is flawed and is of no scientific value.

2. Consumer preference is preference whether it is studied by economists, psychologists, mathematicians, or physicists. Preference under any name, including utility, value, "wants and desires," tastes, or ophelimity is preference (see Section 3.9.4 of [2] for a detailed discussion of this issue). The notion of a different kind of preference for demand theory has no logical basis.

3. The mathematical theory of utility which is relevant to the traditional theory of consumer demand is subject to the same mathematical principles that apply to any other mathematical theory. The fact that there is widespread acceptance among economists of older incorrect notions of the mathematical theory of utility indicates an urgent need to correct these notions. Errors whose acceptance is widespread need to be corrected rather than defended.

4. The marginal utilities which are relevant to the traditional theory of consumer demand are partial derivatives of utility functions. Derivatives are concepts of differential calculus. There is no second notion of differential calculus which is the relevant one for the traditional theory of consumer demand. Elementary calculus errors by Hicks, Samuelson, and their followers are analyzed in detail in [2 and 3]. Katzner is defending the widespread misapplication of differential calculus in the traditional theory of consumer demand.

2. More on Hicks's and Samuelson's errors

The operations of calculus, including differentiation, are carried out in a vector space (see e.g. Dieudonne [4]). Vector spaces and the operations of calculus are quantitative concepts (for formal definitions see [2, §3.7]). It follows that the notion of "non-quantitative calculus" is a contradiction in terms yet, according to Hicks [5, p. 19], the operation of differentiation is applicable on utility functions that he has "purged" from quantitative concepts. Hicks and all economists who reject "all concepts which are tainted by quantitative utility" are rejecting the application of calculus in utility theory and thereby the very concept of marginal utility. Calculus is quantitative, differentiation is quantitative, and marginal utility, which is a derivative, *is* a quantitative concept. The notion of non-quantitative differentiation is unique to microeconomics.

The Hicksian purge applies to *all* concepts of quantitative utility of *any* kind. Furthermore, there is no support for Katzner's second kind of ordinal utility in the literature. Repeating Hicks's ordinal utility error, Samuelson correctly gives *the only possible definition of the only kind of ordinal utility* in Equations (6-8) of [12, p. 94] which he verbally describes [12, p. 91] as "*ordinal* preference, involving "more" or "less" but not "how much," but then he incorrectly claims that only ordinal preference is required for the analysis of consumer's behavior. Also note Samuelson's use of *preference* as synonymous to *utility*.

In addition, Hicks [5], Samuelson [12], Mas-Colell et al. [9], and all authors who claim that ordinal information is sufficient for the existence of utility derivatives, including Katzner [6], rely on an incorrect application of the Implicit Function Theorem of calculus. The onus is on these authors to establish that the assumptions of this theorem (see e.g. Apostol [1, p. 147]) are satisfied. Applying the Implicit Function Theorem where the conditions for its applicability are not satisfied is an elementary error in consumer demand theory.

Samuelson correctly says (see [12, p. 94, Equation (9)]) that any monotone increasing transformation of an ordinal utility function is an equivalent ordinal utility function, but the rest of his argument fails on the infinitely many non-differentiable monotone increasing functions that cannot be differentiated. Differentiating these non-differentiable transformations is an elementary error. Moreover, Samuelson's faulty argument applies verbatim to the case where

no ordinal information on the indifference surfaces is available (the numerical value of the utility of x equals the numerical value of the utility of y if and only if the consumer is indifferent between x and y). This implies the absurd claim that the quantitative tools of differential calculus apply on utility functions where the only available information is whether $u(x)$ does or does not equal $u(y)$. The ordinal utility claim, which is based on the same errors, is just as absurd. The notion of differentiating ordinal functions has no counterpart in science — vector space operations are not applicable on ordinal data and ordinal functions are not differentiable. Physics (and mathematics) should be rewritten if ordinal information is sufficient for the application of differential calculus.

Finally, if the partial derivative of a utility function with respect to one of its variables does not exist, the assumptions of the Implicit Function Theorem are not satisfied and this theorem cannot be employed to "prove" that although the derivatives do not exist, their ratios do exist (cf. Hicks [5, p.19]). This, too, is an error.

3. Summary

The claim that there is a second notion of ordinal utility on which a second kind of mathematics applies is untenable. Katzner cannot ignore the fundamental issue of the conditions for applicability of the operations of algebra and calculus. These operations cannot be applied where the conditions for applying them are not satisfied. He cannot ignore the counter-examples and the detailed analysis of the errors committed by Hicks and Samuelson in Section 3.4 of [2]. And he must show that the conditions for applying the Implicit Function Theorem are satisfied where they are used in demand theory. Demand theory's errors should be corrected, not defended.

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