Constant returns to scale: can the market economy exist?

M. Shahid Alam[1]   [Northeastern University, USA]

Abstract
This note argues that the competitive paradigm of neoclassical economics breaks down in the presence of constant returns to scale (CRS). With CRS, all goods can be produced at identical unit costs by all economic agents, making self-production a feasible alternative to market production. In the event, an infinite number of equilibria become possible with a mix of markets and self-production. If labor is the only factor of production, self-production is the only option and the market economy ceases to exist.

All too often the competitive paradigm of neoclassical economics has been faulted for its unrealistic assumptions, but rarely for logical inconsistency. These critics have also developed several new variants of economics based on more realistic assumptions.[2] Nevertheless, these critiques have failed to dislodge the competitive paradigm from its preeminent position in economics. Kenneth Arrow (1994: 451) has argued that this is because the competitive paradigm "is still the only coherent account of the entire economy." Others point to the ideological function of neoclassical economics: in particular, its defense of the capitalism of unfettered markets.[3]

This note presents a different critique: it shows that the competitive paradigm is inconsistent with constant returns to scale (CRS). At least since Leon Walras, economists have constructed mathematical models to establish the exact conditions under which a market economy – with fixed resources, tastes and technology – can attain an equilibrium that is also Pareto-efficient. CRS – or the more general assumption of non-increasing returns to scale – has been a cornerstone of all these models. Allyn Young (1928) and later Nicholas Kaldor (1967 and 1972) have shown that equilibrium economics becomes irrelevant in the presence of increasing returns to scale. We argue that neoclassical economics is in trouble even *with* CRS: and this is a problem that has gone unnoticed. Our concern is not with the existence – or uniqueness and stability – of solutions to the system of equations that define the neoclassical economy. Instead, we ask a more basic question. Can a *market* economy exist in the presence of CRS?

A short note might suffice to answer this question, but lest it become too short we will start with the mercantilists and Adam Smith. This digression will provide some historical

---

[1] The author wishes to acknowledge comments on this paper from Salim Rashid, Professor Emeritus at University of Illinois, Urbana-Champaign; James Dana, Professor of Economics and Strategy, Northeastern University; Bill Dickens, University Distinguished Professor, Northeastern University; and Asad Zaman, Professor of Economics, International Islamic University, Islamabad. The usual disclaimer applies: only I am responsible for the contents of this paper.

[2] A list of these new approaches might include the old and new institutional economics, behavioral economics, the economics of limited information, evolutionary economics, and the applications of game theory to the study of strategic behavior.

[3] In his Nobel lecture, Joseph Stiglitz (2001) writes that "one cannot ignore the possibility that the survival of the [neoclassical] paradigm was partly because the belief in that paradigm, and the policy prescriptions, has served certain interests." Others explain its ascendancy in the late nineteenth century as a reaction to the radical ideas of Karl Marx. For instance, James Tobin (1985: 30-31) writes, "In positive as well as normative theory, neoclassical economics was in a much better position than classical economics to respond to the Marxist challenge."

perspective on how CRS became one of the cornerstones of neoclassical economics. A little history cannot hurt even in a discipline that does not take too kindly to history.

1. Classical economics

A central purpose of Adam Smith in writing the *Wealth of Nations* was to make the case for laissez faire and, thereby, discredit the mercantilists. It is therefore ironic that he should open his treatise on political economy with a central insight from mercantilists regarding the power of increasing returns to scale in manufacturing.

"The greatest improvement," writes Adam Smith, "in the productive powers of labor, and the greater part of the skill, dexterity, and judgment with which it is anywhere directed, or applied, seem to have been the effects of the division of labor." Division of labor contributes to labor productivity in at least four ways: augmenting worker skills, saving time lost in moving from one task to another, increasing use of machines, and reducing the time they are idle.[4] Perhaps, the increasing use of machinery is the most important source of productivity gains that flow from division of labor: it "enable[s] one man to do the work of many." In pin manufacturing, according to Adam Smith, the average output of each man - working as part of a team of ten workers - is 4,800 pins per day; working independently "they certainly could not each of them have made twenty, perhaps not one pin in a day."[5] The returns to scale in pin-making are quite extraordinary: division of labor increased the output of each worker by a multiple of 240. It is worth noting that these gains were attained without the use of any external sources of energy.

Adam Smith makes it clear that these productivity gains in pin making were not exceptional. "In every other art and manufacture," he writes, "the effects of the division of labor are similar to what they are in this trifling one; though, in many of them, the labor can neither be so much subdivided, nor reduced to so great a simplicity of operation. The division of labor, however, so far as it can be introduced, occasions, in every art, a proportionate increase in the productive powers of labor." Agriculture did not benefit from these productivity gains; the division of labor in this sector was limited by the sequential nature of many of its operations.[6]
Although he does not elaborate, Adam Smith clearly grasped the dynamic implications of the division of labor. "As it is the power of exchanging," he writes in chapter three of his treatise, "that gives occasion to the division of labor, so the extent of this division must always be limited by the extent of that power, or, in other words, by the extent of the market."[7] If a two-way feedback existed between the division of labor and the size of markets, this was likely to create a virtuous circle of growth. Moreover, since the division of labor operated more strongly in manufactures than in agriculture, this necessarily made the former the engine of growth. This is not where Adam Smith wanted to go, however, since it led straight into mercantilist territory.[8]

Having started on the wrong foot, Adam Smith quickly gave up further talk of division of labor.

---

[4] Adams Smith identified the first three.
[5] Adam Smith (1776/1975): 3,7.
[6] Smith (1776/1975): 5-6.
[7] Smith (1776/1975): 17. Allyn Young (1928: 529) has written that this was "one of the most illuminating and fruitful generalizations which can be found anywhere in the whole literature of economics."
[8] Antonio Serra (2011: 121), an Italian writer of the early seventeenth century writes, "In manufacturing, production can be multiplied not merely twofold but a hundredfold, and *at a proportionately lower cost* (emphasis added)."

This was not the foundation on which he could build his critique of dirigisme; he would have to change his focus.[9] And this he did in a hurry. After spending the first three chapters of *The Wealth of Nations* discussing foreign trade, size of markets, productive powers and the division of labor, in chapter four he fixes his attention on price theory and the allocation of resources in free markets. The division of labor would receive only a few passing mentions in the rest of the *Wealth of Nations*.[10]

Adam Smith sought the superiority of markets in their allocative function. Motivated only by self-interest, and guided by market prices, buyers and sellers vary the amounts they buy and sell until each market converges upon its "central price to which the prices of all commodities are continually gravitating."[11] The concept of a central or "natural price" was rooted in CRS.[12] John Hicks (1989: 10) writes that in spite of what Adam Smith has to say about scale economies in his theory of growth, "we still find that in his value theory, his cost of production value theory, he does not get away from CRS." In the presence of increasing returns to scale, output and price changes in one market would be transmitted to other markets through forward and backward linkages making the concept of an equilibrium and a natural price irrelevant. The die was cast: the 'invisible hand' would have to lean on the crutch of CRS.

Adam Smith offered at least three arguments in favor of free trade. It augmented markets, thus giving impetus to growth *via* division of labor; it allowed capital to flow into the most productive channels based on a country's absolute advantage; and it gave vent to the country's surplus. The first argument was problematic; since it reinforced existing advantages in manufactures and commerce it would disadvantage countries that entered free trade with a handicap in these sectors. The second argument would work only if capital *or* labor was completely free to cross borders.[13] The third argument offered free trade as a remedy for the surpluses created by shifts in trade away from a country' exports; but greater exposure to trade might also worsen the problem of surpluses. In other words, the *Wealth of Nations*, failed to make a strong case for free trade. This deficiency had to be remedied.

It is believed that David Ricardo (1817: 93-95) brilliantly rose to the occasion with his theory of comparative advantage. His success however came at a high price: he examines trade within a purely static framework. For four centuries, the mercantilists had framed their arguments for protectionism within a dynamic framework: they argued that a lagging country was unlikely to improve its chances of growth or preserve its sovereignty under free trade. Ricardo framed his question within a purely static framework: he fixed technology, tastes and labor in each country. He makes another critical assumption: production is subject to CRS. Within this static framework, free trade is always the best policy whether a country's comparative advantage lies in potato chips or computer chips.

This shift towards static analysis was an ideological necessity. Britain had outgrown the mercantilist policies that had elevated it from the ranks of a backward country in the early fifteenth century to become the world's leading economy by the mid-eighteenth century. In an

---

[9] In the middle of chapter four of the *Wealth of Nations*, writes Nicholas Kaldor (1972/1989, 378), Adam Smith "suddenly gets fascinated by the distinction between money price, real price and exchange value, and from then on, hey presto, his interest gets bogged down in the question of how values and prices are determined for products and factors of production."

[10] Smith (1776/1975): 64, 84,243-44,415-416, 659, and 707. In only two places (415-16 and 706), Adam Smith briefly touches upon the connection between size of markets and division of labor.

[11] Smith (1776/1975): 58.

[12] For his discussion of the relationship between market price and the natural price, see Smith (1776/1975): Book I, chapter 7.

[13] Alam (2000: 50-52) deals with the first contradiction.

essay on the rich country-poor country debate among Scottish thinkers in the eighteenth-century, J. M. Low (1952, 324) writes that this was "the central point of the *Wealth of Nations*." "Smith and [Josiah] Tucker," he continues, "were agreed that Britain was already a rich country and hence that there really was no need for the government to intervene to safeguard advantages which we were in no danger of losing." Unlike Adam Smith, however, Josiah Tucker had the candor to concede that what was good policy for Britain was not good for poor countries. Protectionism would be the right policy for the lagging countries, since only this could raise productivity in their manufacturing activities.

2. Neoclassical economists

Once the classical economists had chosen to demonstrate the superiority of free markets within the static framework of allocative efficiency, the path was clear for the marginalist revolution of the 1870s with its focus on methodological individualism and marginalist analysis.[14]

In order to demonstrate the efficiency of markets in equilibrium, economic theory would have to show that the decisions of self-interested buyers and sellers led to equilibrium in all markets that also produced a Pareto-efficient allocation of resources.[15] Leon Walras formally launched this quest in his *Elements of Pure Economics*, published in two parts in 1874 and 1877. Crucially, his general equilibrium analysis assumed CRS in production: in addition, capital, labor and land always entered his production function with fixed coefficients. Employing more rigorous mathematics, Kenneth Arrow and Gérard Debreu (1954) and Gérard Debreu (1959) demonstrated the existence of a unique solution to their system of equations; they also established the two fundamental theorems of welfare economics. Their general equilibrium system too was built on the assumption of non-increasing returns to scale.

The neoclassical production function with smooth factor substitution and CRS underpins at least two additional ventures in neoclassical economics. Although the concept of marginal product of labor had been around since the early nineteenth century, the marginal productivity theory of factor prices was developed only in the late nineteenth century. While earlier writers had spoken of the law of diminishing returns in agriculture, in 1888 John Bates Clark developed a nearly full-blown theory of factor prices rooted in the law of diminishing returns. He claimed that this law applies to the marginal return to *any* homogenous factor when it is combined with fixed quantities of other factors, provided the technology of production remains fixed. For this theory to work, however, payments to factors would have to fully exhaust the total product. In 1894, Philip Wicksteed (1931) pointed out that product exhaustion could occur only in the presence of CRS. Some five decades later, George Stigler (1941: 49) added another

---

[14] The Austrian economists did not think in terms of equilibrium. In neoclassical economics, individuals have perfect information and tastes and technology are given; discovering equilibrium in this system is a problem in constrained optimization since consumers maximize utility and producers maximize profits. Individuals in the Austrian system possess only local information about matters that concern them; and only individuals possess information about their tastes and the resources and technology that are available to them. As a result, individuals alone have the best chance of making the right decisions in the face of changing tastes, technology and prices. Moreover, this is a constantly evolving system since the information on which individuals base their decisions are constantly changing. In other words, economic life consists in the discovery of new and better information in a world of changing information.

[15] Other factors too may have propelled the timing of this shift or the mathematization that accompanied it. In part, at least, it would appear that the marginalist revolution was advanced to counteract the revolutionary twist Karl Marx gave to the labor theory of value. Philip Mirowski (1990: chapter 7) has written about the physics envy that pushed some economists in the late nineteenth century to adopt marginalist analysis and mathematical formulae.

twist: the law of diminishing returns itself may not hold in the presence of increasing returns to scale. In other words, without CRS the neoclassical theory of production would have to abandoned.

3.   Can the neoclassical economy exist?

The competitive paradigm is the capstone in the edifice of mainstream economics. Every graduate student learns, whether he comprehends it or not, that the system of equations that defines this paradigm possesses a unique equilibrium solution that is also stable and Pareto-efficient. However, soon after this paradigm was completed in the 1950s, a variety of critics began to point out that this system cannot stand up to changes in any of its numerous assumptions.

Building on the work of Allyn Young (1928), Nicholas Kaldor (1972) demonstrated the irrelevance of this paradigm in the presence of increasing returns to scale. John Hicks (1989: 16) offered a more friendly critique. He concluded that "if used with proper precaution in the approximate contexts, [CRS] can be a help: but if misused, or if it is applied in the wrong context, it can indeed, as Kaldor thought, be a hindrance." Stiglitz (2001) and his associates have shown that banks cannot and do not rely on market clearing to allocate their funds: instead, they ration their loans. A whole school of behavioral economics has been taking aim at the claims of rationality made by the competitive paradigm.[16]

This note presents a different critique. It shows that the competitive paradigm breaks down in the presence of CRS. First, it shows that under CRS, production of goods for self-consumption will partially or completely displace production for markets. Further, if we assume with David Ricardo that labor is the only factor of production, a natural economy will necessarily and fully replace the competitive market economy. This can be established intuitively.

The presence of CRS in the competitive paradigm means that unit costs of producing goods are fixed regardless of the scale of production. In turn, this means that no advantage can accrue to individual economic agents – all of whom have access to the same technology – from specializing in the production of any good.[17] As a result, each person can supply himself with the goods that he needs as cheaply as anyone else however small the quantities of goods that he needs for his own consumption. All this establishes a presumption that a market economy may emerge in the presence of CRS only when a person's endowment of factors prevents him from producing his preferred consumption bundle.

First, consider the competitive economy as it is. In this economy, x_{1i}^* and x_{2i}^* denote individual i's optimal consumption bundles in a competitive economy that produces two goods, x_1 and x_2. In the competitive paradigm the individual i attains his optimal consumption bundle by selling all his factor endowments on the market and using the proceeds to buy his optimal consumption bundle. However, this is not the only way that an individual can attain his optimal consumption bundle. Under constant unit costs an individual may choose to avoid the market and instead self-produce his consumption bundle. This would be quite straightforward if his endowment of factors turned out to be identical to the bundle of factor services required to produce x_{1i}^* and x_{2i}^*. If the factor endowments of all individuals coincide with the factor

---

[16] See Heukelom (2012).
[17] In the Ricardian model, countries will specialize (and gain from trade) because technology or production is not the same across countries.

requirements for their consumption bundles, production for the market becomes unnecessary. Individuals in this economy would be indifferent between self-producing their consumption bundles or buying them on the market. It might still be possible to preserve the market economy if we assumed that *all* individuals prefer market-acquired consumption bundles over self-produced ones. On the other hand, the existence of a market economy would be ruled out if we introduce transaction costs into the model.

It is much more likely, however, that an individual may not possess exactly the factor endowments that are required to produce his optimal consumption bundle. He may overcome this mismatch between the two bundles *via* different equivalent exchanges: trading goods, trading factors, or some combination of the two. In the absence of transaction costs - another assumption of the neoclassical model – individuals will be indifferent among the three types of trades. An individual could use his factor endowment to produce goods and meet the deficit between this production bundle and his optimal consumption bundle *via* trading on the markets for goods. Alternatively, he may use *some* part of his factor endowments for self-production and trade both goods and factors to attain his desired consumption bundle. As a result, an infinite number of options exist for each individual to attain his consumption objective. If all individuals chose to trade *all* of their factors this and this alone would reproduce the competitive equilibrium of neoclassical economics. But this is a limiting case: one equilibrium amongst an infinite number of possible equilibria. In all other scenarios, with at least *some* individuals engaging in *some* amount of self-production, the economy would combine markets and self-production to satisfy the demands of consumers.

If labor is the only factor of production, CRS will rule out all market exchanges. Since labor is the only factor of production, each individual can produce *any* bundle of goods so long as this satisfies his labor constraint. If x_1 and x_2 are two goods in this economy, and a_{L1} and a_{L2} are the corresponding labor coefficients, the individual's production function is given by

(1) $x_1.a_{L1} + x_2.\ a_{L2\ =}\ L,$

where L is his endowment of labor. Equation (1) also represents the individual's budget constraint. In other words, every individual can self-produce *any* point he chooses on his budget constraint because the budget constraint is also his production function. Since production and consumption bundles in this economy are identical for *all* individuals, there is no need for trade. Trade would arise only if production technology – the labor coefficients for different goods – varied across individuals. Since the competitive paradigm rules this out, the neoclassical market economy cannot exist.

4.   Some concluding observations

How did neoclassical economists miss this inconsistency in their core model? Perhaps the answer is to be found in the ideological function of mainstream economics: many of its most avid practitioners have used it to demonstrate the superiority of free market capitalism.

It was Adam Smith who started economics on this course. Aware that a dynamic analysis of markets could just as easily support dirigisme, he shifted gear and chose to make the case for free markets within a static framework with constant unit costs. Following his lead, David Ricardo formally established the superiority of free trade with constant costs. However, no formal proof yet existed of the superiority of free markets. Three developments in the late

nineteenth century would advance economists towards this goal: the marginalist revolution, the acceptance of Vilfredo Pareto's definition of efficiency, and the use of mathematical tools borrowed from physics and engineering. Even so it took a while to work out the exact specifications of a decentralized economy that could play host to the invisible hand. Kenneth Arrow and Gerard Debreu finally reached this Valhalla in the 1950s: or so they thought.[18] The intellectual gymnastics this required quite staggers the sober mind not mesmerized by the mathematical beauty of general equilibrium economics. The assumption of CRS was just one of several tricks in the acrobatics of neoclassical trapeze artists.

Could economists have developed a defense of free markets on less restrictive assumptions? The Austrian economists claim that they have done so. Nevertheless, the Austrians take a distant second place to neoclassical economics in the mainstream discourse on free markets. Why is this the case? The neoclassical emphasis on market equilibria and Pareto-efficiency has an important ideological advantage over the Austrian analysis of markets with its emphasis on informational efficiency. The first lent itself to mathematization, the second did not.[19] In the competition for dominance, therefore, neoclassical economists could claim the prestige of mathematics; this was a handicap for the Austrians. In addition, the mathematization of neoclassical economics aided in the professionalization of economics. On the one hand, it outfitted economists with the "tools" and "models" that gave them the prestige of physics and engineering. Mathematization also proved to be a powerful engine of job-creation: especially since reality could impose no check on the proliferation of economic models. The fate of economics was sealed.

References

Ackerman, Frank and Alejandro Nadal, *The flawed foundations of general equilibrium: Critical essays on economic theory* (London and New York: Routledge, 2004).

Alam, M. Shahid, *Poverty from the wealth of nations* (Houndmills, UK and New York: Macmillan and St. Martin's Press, 2000).

Arrow, Kenneth, "Beyond general equilibrium," in: G. A. Cowan, D. Pines and D. Meltzer, eds., *Complexity: Metaphors, models and reality* (Boston, MA: Addison-Wesley, 1994).

Arrow, Kenneth and Gérard Debreu, "Existence of an equilibrium for a competitive economy," *Econometrica* 22 (1954): 265-90.

Arrow, Kenneth, "An extension of the basic theorems of classical welfare economics," in *Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability* (Berkeley: University of California Press, 1951).

Debreu, Gérard, *The theory of value* (New York: Wiley, 1959).

Heukelom, Floris, "Behavior economics," in: John B. Davis and D. Wade Hands, eds., *The Elgar companion to recent economic methodology* (Cheltenham, UK: Edward Elgar, 2012).

Hicks, John, "The assumption of constant returns to scale," *Cambridge Economic Journal* 13 (1989): 9-17.

Kaldor, Nicholas, "The role of increasing returns in industry," in: *Strategic factors in economic development* (Ithaca, NY: Cornell University Press, 1967).

---

[18] Some critics of neoclassical economics cast doubt on this claim: see Frank Ackerman and Alejandro Nadal (2004).

[19] Paul Samuelson may have clinched this trend with his *Foundations of Economic Analysis* published in 1947.

Kaldor, Nicholas, "The irrelevance of equilibrium economics," Economic Journal 82 (December 1972), reprinted in F. Targetti and A. P. Thirlwall, eds., *The essential Kaldor* (New York: Holmes and Meir, 1989).

Low, J. M., "An eighteenth century controversy in the theory of economic progress," *The Manchester School*(20, 3 (September 1952): 311-30.

Mirowski, Philip, *More heat than light: Economics as social physics, physics as nature's economics* (Cambridge: Cambridge University Press, 1990).

Ricardo, David, *Principles of political economy and taxation* (New York: Prometheus Books, 1817/1996).

Samuelson, Paul, *Foundations of economic analysis* (Cambridge, MA: Harvard University Press, 1947).

Serra, Antonio, *A short treatise on the wealth and poverty of nations*, translated by Jonathan Hunt and edited by Sophus A. Reinert (London: Anthem Press, 1613/2011).

Smith, Adam, *An inquiry into the nature and causes of the wealth of nations* (New York: Modern Library, 1776/1975).

Stigler, George J., *Production and distribution theories* (New York: Macmillan, 1941).

Stiglitz, Joseph, *Information and the change in the paradigm of economics* (Stockholm: Prize Lecture, December 8 2001). http://www. nobelprize.org/nobel_prizes/economics/laureates/2001/stiglitz-lecture.pdf

Tobin, James, "Neoclassical theory in America: J. B. Clark and Fisher," *American Economic Review* 75, 6 (December 1985): 28-38.

Young, Allyn, "Increasing returns and economic progress," *Economic Journal* 38, 152 (December 1928): 527-42.

Wicksteed, Philip H., "An essay on the coordination of laws of production," *The Economic Journal*, 4, 14 (June 1931): 303-13.

Author contact: m.alam@neu.edu

_____

You may post and read comments on this paper at http://rwer.wordpress.com/2013/07/02/rwer-issue-64/